Physics 100: Homework Assignment #12

Due on Friday, May 18at the Beginning of Class

Instructor:	Nathan Bramall	**Lecture:**	MWF 9:10 - 10:00
Phone:	(650) 378-7301 x 19227		Room 36-109
Email:	bramalln@smccd.edu	**Office Hours:**	Wednesdays 10-1 pm, 36-105F
Office:	36-105F	**Section:**	Wednesdays, 1-2 pm, 36-112

Course Web Page: http://smcweb.smccd.net/accounts/bramalln/

You may use this sheet for the fill-in-the-blank questions, but please use a separate sheet for the short-answer questions. Remember also to show all of your work on any calculations!

Section 1. **Short Answer Questions (4 pts. each)**

1. In a reference frame that measures the *proper time* between two events, what is the distance between these two events?

2. If a reference frame measures the *proper length* of a rocket, what can be said about the relative velocity between the rocket and the reference frame?

3. Consider a frog jumping on the table of a train and the following two reference frames:

   Frame A: at rest with respect to the Earth
   Frame B: at rest with respect to the train

   For an observer on the train, the frog appears to jump straight up and down, landing in the exact same place where it jumped from, and takes a 2 seconds to do so.
   (A) Which reference frame measures the *proper time* between the events of the frog jumping and the frog landing?
   (B) How long does it take the frog to jump according to **Frame A** and **Frame B** if **Frame B** measures the train's speed to be 0.995 c?

4. **(A)** Consider again the reference frames and train in Problem 3. If the conductor of the train (at rest in **Frame B**) measures the train to be 500 m long, how long will the train be according to the people sitting at the station (at rest in **Frame A**)?
   (B) Which reference frame measures the *proper length* of the train?

5. Consider two reference frames:

   Frame A: at rest with respect to the Earth
   Frame B: at rest with respect to a spaceship flying past the Earth

   (A) If **Frame A** measures the velocity of the spaceship to be +200 m/s, with what velocity does **Frame B** measure for the Earth?
   (B) If **Frame B** measures the velocity of a satellite to be 500 m/s, with what velocity does **Frame A** measure for the satellite?
   (C) Now consider the case where **Frame A** measures the velocity of the spaceship to be 0.75 c. With what velocity does **Frame B** measure for the Earth?
   (D) Now, if **Frame B** measures the velocity of a satellite to be 0.5 c, with what velocity does **Frame A** measure for the satellite?

6. Consider a twin pair o' ducks, one named Filbert and one named Hog. Filbert boards a space ship and travels to Planet Z, which is fixed relative to the Earth– in other words, their relative velocities are zero. Hog uses his rest frame to measure Filbert's speed to be 0.9 c and the distance between Earth and Planet Z to be 10 *light years*[1]. Define the two events of importance as (1) Filbert leaving Earth and (2) Filbert arriving at Planet Z.
   (A) Which duck measures the *proper time* between the two events?
   (B) Which duck measures the *proper distance* between the two planets?
   (C) According to Hog's rest frame, how long does it take Filbert to travel to Planet X and how far away is Planet X?
   (D) According to Filbert's rest frame, how long does it take him to travel to Planet X and how far away is Planet X?
   (E) Which duck's measurements are correct?

7. If you add 100 J of energy to a 1 kg brick, by how much does its mass change?

8. Consider Filbert's spaceship from Problem 6. Assume it has a mass of 10,000 kg.
   (A) If Hog were using Newtonian ideas, what would the spaceship's momentum be according to his rest frame?
   (B) If Hog were using special relativity, what would the spaceship's momentum be according to his rest frame?

---

[1] A *light year* is a measure of distance: it is the distance light travels in the time of one year. In other words it is 1 $year \times c$. For this problem it is easiest to use units of years. For example, you can use the equation $time = distance/speed$ to figure out how long it takes to travel 5 *light years* while traveling at a speed of 0.5 c:

$$time = \frac{5\ light\ years}{0.5\ c} = \frac{5\ c\ years}{0.5\ c} = \frac{5\ \not{c}\ years}{0.5\ \not{c}} = \frac{5\ years}{0.5} = 10\ years$$

Physics 100: **Solutions** to Homework Assignment #12

Was Due on Friday, May 18at the Beginning of Class

Section 1. **Short Answer Questions (4 pts. each)**

1. In a reference frame that measures the *proper time* between two events, what is the distance between these two events?

   Answer: If a reference frame measures the *proper time* between two events, those two event have to occur in the same location. Therefore the distance between two events in the frame is zero, zip, ol' 0, nada, nichts.

2. If a reference frame measures the *proper length* of a rocket, what can be said about the relative velocity between the rocket and the reference frame?

   Answer: If a reference is measuring the *proper length* of an object, then that object must be at rest in that reference frame. Therefore, the relative speed between a rocket and a reference frame that measures the *proper length* of the rocket is zero, zip, ol' 0, nada, nichts.

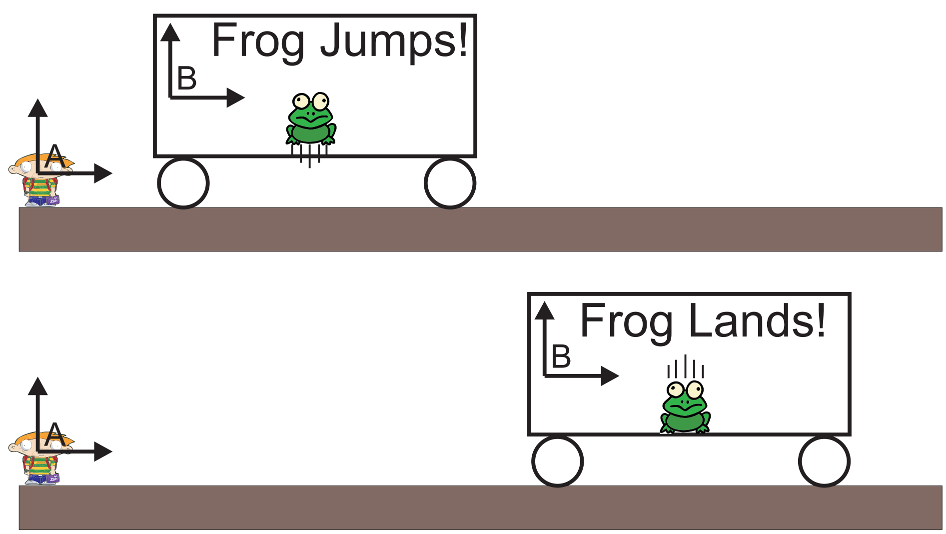
3. Consider a frog jumping on the table of a train and the following two reference frames:

   Frame A: at rest with respect to the Earth
   Frame B: at rest with respect to the train

   For an observer on the train, the frog appears to jump straight up and down, landing in the exact same place where it jumped from, and takes a 2 seconds to do so.
   (A) Which reference frame measures the *proper time* between the events of the frog jumping and the frog landing?
   (B) How long does it take the frog to jump according to **Frame A** and **Frame B** if **Frame B** measures the train's speed to be 0.995 c?

   Answer:

   (A) The *proper time* between two events is measured by the reference frame in which the two events happen *in the same place*. Shown below are the two events and the reference frames:



According to **Frame A**, the frog jumped and landed in two different places[2], so **Frame A** did not measure the proper time between the two events. In **Frame B**, however, the frog jumped and landed in the same place[3] (remember **Frame B** is stuck to the train– the measurements **Frame B** makes are all referenced to a point that moves with the train). Therefore **Frame B** measured the proper time between the two events.
(B) We know that **Frame B** measures the proper time, $\Delta T_\circ$, between these two events, so we know that the proper time is 2 s. We can use the time dilation equation to find ΔT, the time between these events is according to **Frame A**:

$$\Delta T = \gamma \Delta T_\circ$$

Now, recall the definition of γ:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In this case:

$$\gamma = \frac{1}{\sqrt{1 - \frac{(0.995\ c)^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.990}} = \frac{1}{0.1} = 10$$

Therefore, we know that, according to the Earth-based reference frame,

$$\Delta T = \gamma \Delta T_\circ = 10 \times 2\ s = 20\ s$$

4. **(A)** Consider again the reference frames and train in Problem 3. If the conductor of the train (at rest in **Frame B**) measures the train to be 500 m long, how long will the train be according to the people sitting at the station (at rest in **Frame A**)?
(B) Which reference frame measures the *proper length* of the train?

Answer:

(A) The proper length of something is measured by a reference frame in which that something is at rest. Therefore, the reference frame of the conductor measures the proper length of the train she is on. In this case, we know that the proper length, $L_\circ = 500\ m$. We can use the length contraction equation to find out what the Earth-based reference frame measures the length of the train to be:

$$L = \frac{L_\circ}{\gamma}$$

Since the relative speed between the train and the Earth hasn't changed from Problem 3, γ remains unchanged– $\gamma = 10$. Putting all the numbers in:

$$L = \frac{L_\circ}{\gamma} = \frac{500\ m}{10} = 50\ m$$

(B) Well, we already figured out that **(B)** measures the proper length of the train.

5. Consider two reference frames:

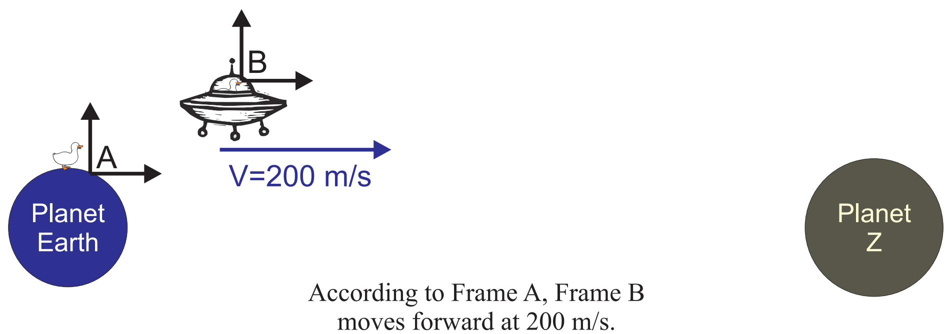
Frame A: at rest with respect to the Earth
Frame B: at rest with respect to a spaceship flying past the Earth

(A) If **Frame A** measures the velocity of the spaceship to be +200 m/s, with what velocity does **Frame B** measure for the Earth?
(B) If **Frame B** measures the velocity of a satellite to be 500 m/s, with what velocity does **Frame A**

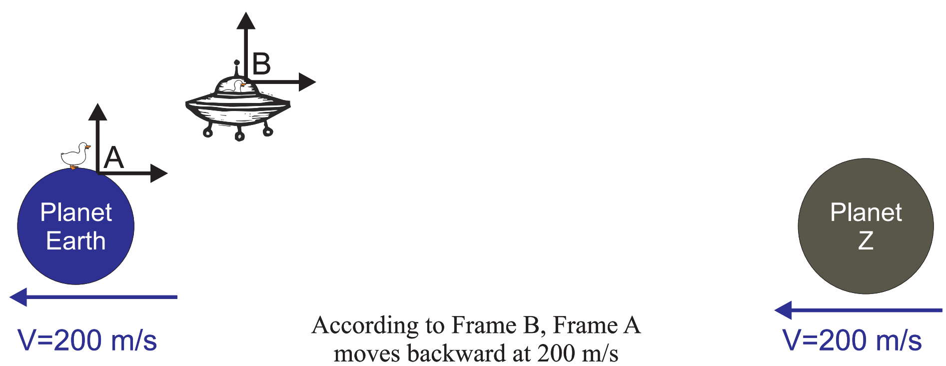
[2] This is obvious because the locations of the jumping frog and the landing frog, relative to the origin of **Frame A** are different.
[3] This is obvious because the locations of the jumping frog and the landing frog, relative to the origin of **Frame B** are the same.

measure for the satellite?
(C) Now consider the case where **Frame A** measures the velocity of the spaceship to be 0.75 c. With what velocity does **Frame B** measure for the Earth?
(D) Now, if **Frame B** measures the velocity of a satellite to be 0.5 c, with what velocity does **Frame A** measure for the satellite?

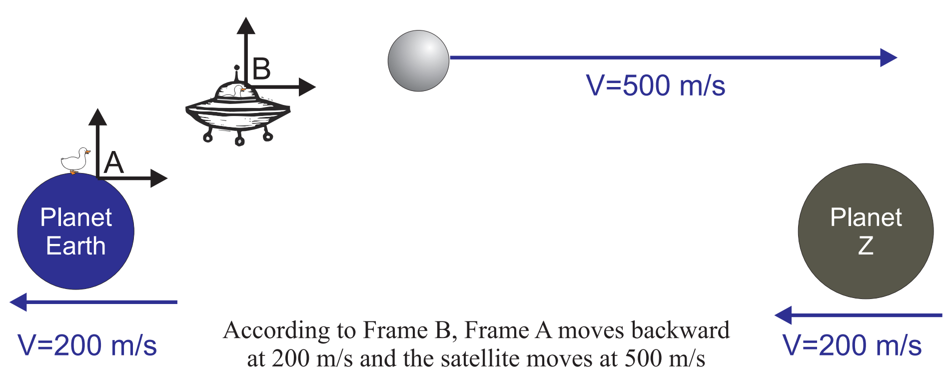
Answer: vskip3mm **(A)** According to Frame A, Frame B moves forward at 200 m/s. This is shown below:



According to Frame B, Frame A moves *backward* at 200 m/s. This is shown below:



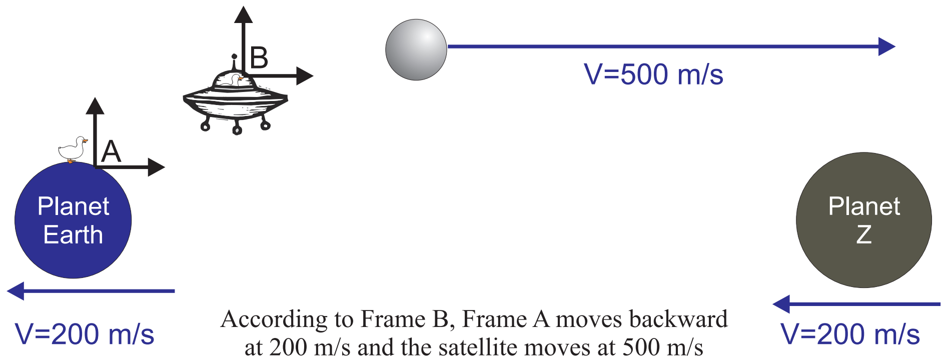
(B) **Frame B** now measures the speed of a satellite to be 500 m/s, as shown below:



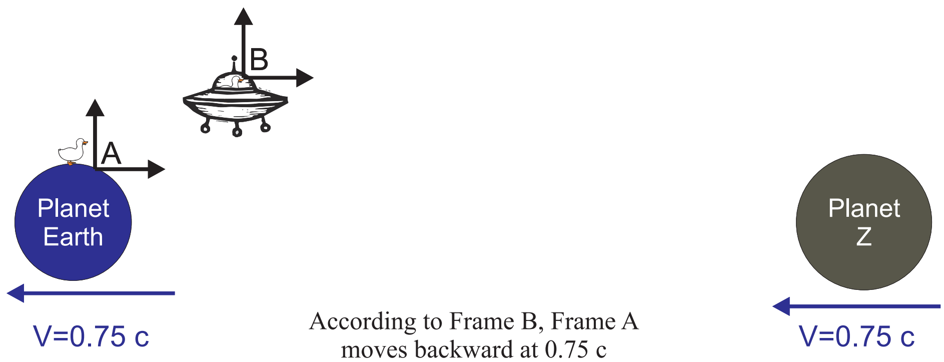
What will **Frame A** measure for the speed of the satellite? Well, the speeds involved here are all very small compared to the speed of light (300,000,000 m/s), so Newtonian Relativity (what we're all used to, at heat) applies. Therefore, we find that the speed of the satellite, according to **Frame A** is simply the addition of the speed that **Frame B** measures for the satellite and the speed that **Frame A** measures for **Frame B**:

$$V_{sat,A} = V_{sat,B} + V_{B,A} = 500\ m/s + 200\ m/s = 700\ m/s$$

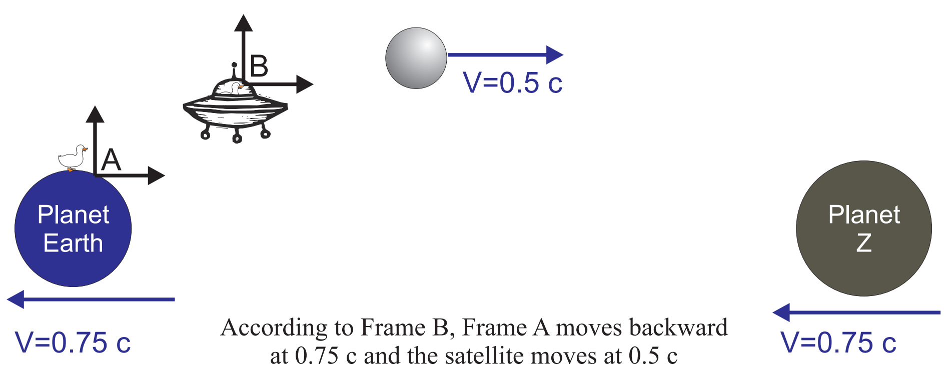
So **Frame A** sees the satellite moving at 700 m/s, as shown below:



(C) Although the speeds have become significant, relative to the speed of light, the relative speed that one reference frame measures for another reference frame remains unchanged. In this case, **Frame B** measures the velocity of the Earth to be 0.75 c backwards, as shown below:



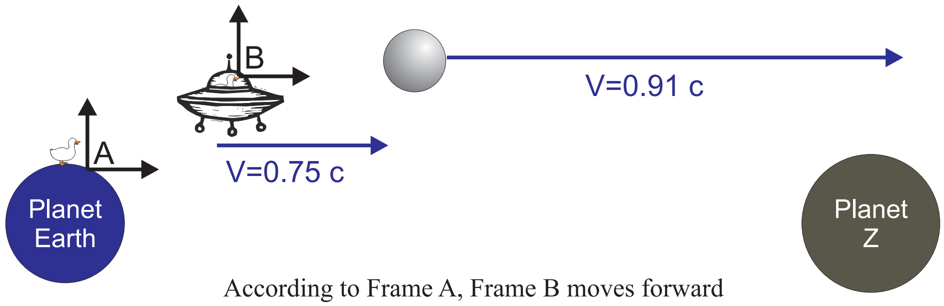
(D) The set-up of this problem is very much like part **(B)**:



Since the speeds are so much higher, we cannot use Newtonian Relativity, but must use Einstein's Special Relativity to figure out what the velocity of the satellite is according the **Frame A**:

$$V_{sat,A} = \frac{V_{sat,B} + V_{B,A}}{1 + \frac{V_{sat,B}\, V_{B,A}}{c^2}}$$

$$V_{sat,A} = \frac{0.5\ c + 0.75\ c}{1 + \frac{0.5 \times 0.75\ c^2}{c^2}} = \frac{1.25\ c}{1 + 0.375} = 0.91\ c$$



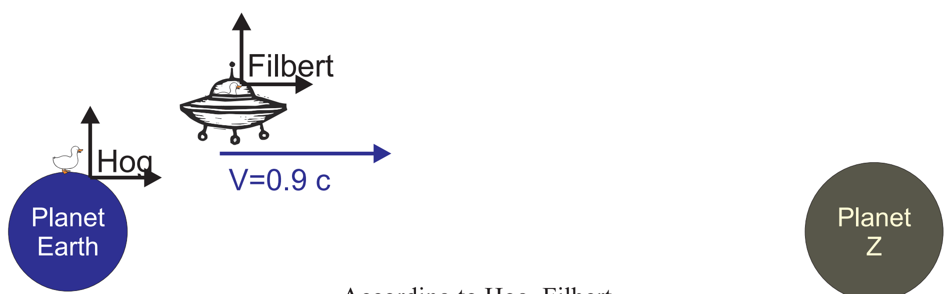
According to Frame A, Frame B moves forward
at 0.75 c and the satellite moves at 0.91 c.

So **Frame A** measures the speed of the satellite to be 0.91 c, as shown in the above diagram. Note that if we used Newtonian Relativity, we would have determined that the speed **Frame A** measures for the satellite would be 1.25 c– faster than the speed of light, and significantly faster than reality!

6. Consider a twin pair o' ducks, one named Filbert and one named Hog. Filbert boards a space ship and travels to Planet Z, which is fixed relative to the Earth– in other words, their relative velocities are zero. Hog uses his rest frame to measure Filbert's speed to be 0.9 c and the distance between Earth and Planet Z to be 10 *light years*[4]. Define the two events of importance as (1) Filbert leaving Earth and (2) Filbert arriving at Planet Z.
(A) Which duck measures the *proper time* between the two events?
(B) Which duck measures the *proper distance* between the two planets?
(C) According to Hog's rest frame, how long does it take Filbert to travel to Planet X and how far away is Planet X?
(D) According to Filbert's rest frame, how long does it take him to travel to Planet X and how far away is Planet X?
(E) Which duck's measurements are correct?

Answer:

Before delving into the sections of the problem, let's just establish what is going on from both Hog's and Filbert's point of view. According to Hog, Filbert is moving forward at 0.9 c while the Earth and Planet Z remain at rest:

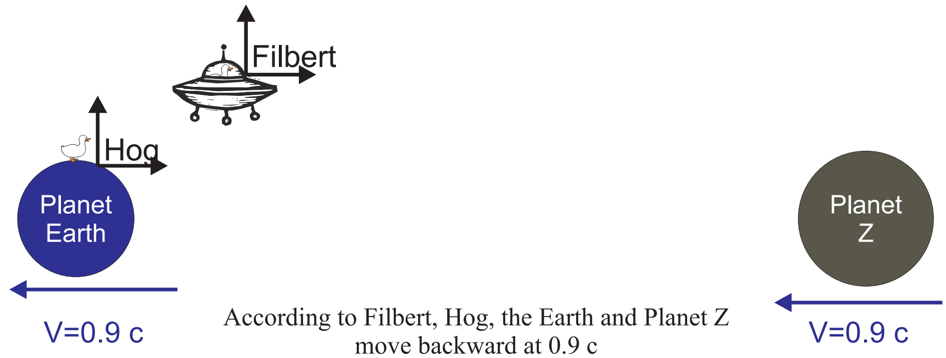


According to Hog, Filbert
moves forward at 0.9 c while the Earth and Planet Z sit still

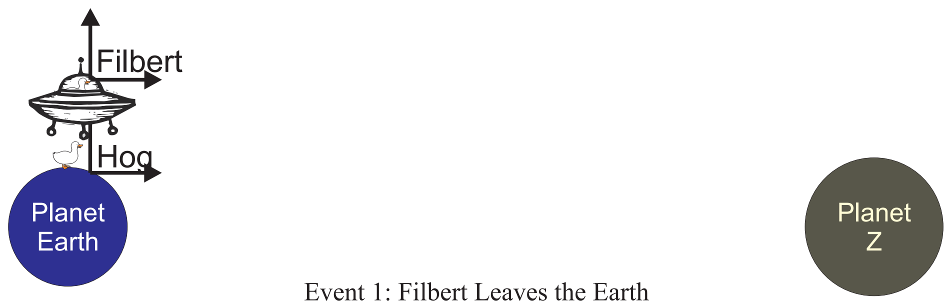
According to Filbert, Hog, the Earth, and Planet Z are all moving backwards at 0.9 c

[4]A *light year* is a measure of distance: it is the distance light travels in the time of one year. In other words it is 1 *year* × c. For this problem it is easiest to use units of years. For example, you can use the equation $time = distance/speed$ to figure out how long it takes to travel 5 *light years* while traveling at a speed of 0.5 c:

$$time = \frac{5\ light\ years}{0.5\ c} = \frac{5\ c\ years}{0.5\ c} = \frac{5\ \cancel{c}\ years}{0.5\ \cancel{c}} = \frac{5\ years}{0.5} = 10\ years$$



(A) Which duck measures the *proper time* between the two events? Let's look at the two events:

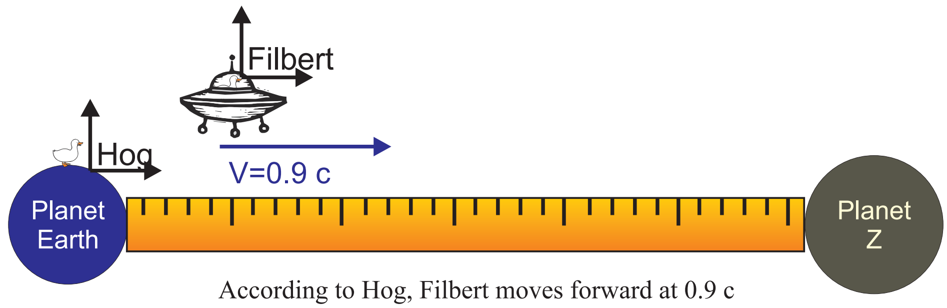


Event 1: Filbert Leaves the Earth

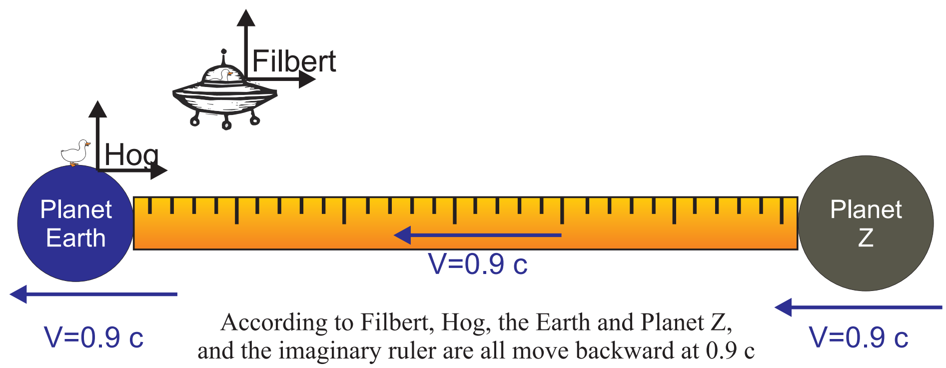
Event 1: Filbert Arrives at Planet Z

According to Hog, the two events happened 10*light years* away, so he doesn't measure the proper time between the events. According to Filbert, the two events happened at the same place: right outside of his spaceship[5]! Therefore Filbert measures the proper time between events (1) and (2).
(B) The proper length of an object is measured by the reference frame in which that object is at rest. Likewise the proper distance between two objects is measured by the reference frame in which those two objects are both at rest. If the second sentence causes you a bit of trouble, but you're OK with the first, imagine that Filbert and Hog measure the distance between planets using a giant ruler, the length of which gives the distance between the planets. According to Hog, the ruler is at rest, so he definitely measures the proper length. According to Filbert, however, the ruler is moving backward at 0.9 along with Hog, the Earth, and Planet Z:

[5]If this is not obvious, remember that the distance something happens is given by the distance from the origins of the little axes that I drew for both Filberts and Hog's reference frames. For Filbert, the distance to the Earth at Event 1 is the same as the distance to Planet Z at Event 2– therefore they happen in the same place, according to Filbert's frame.



According to Hog, Filbert moves forward at 0.9 c
while the Earth, Planet Z, and the imaginary ruler sit still



According to Filbert, Hog, the Earth and Planet Z,
and the imaginary ruler are all move backward at 0.9 c

(C) According to Hog's rest frame, between those two events, Filbert travels a distance of 10 *light years* at a rate of 0.9 *c*. Using the $time = distance/rate$ equation, we find:

$$\Delta \mathrm{T}_{Hog} = \frac{10\ light\ years}{0.9\ c} = \frac{10\ c\ years}{0.9\ c} = \frac{10\ years}{0.9} = 11.1\bar{1}\ years$$

(D) Since we know what Hog measured for the time, and we know that Filbert measures the proper time, we can use the time dilation equation to figure out exactly what Filbert measured:

$$\Delta \mathrm{T} = \gamma \Delta \mathrm{T}_{\circ}$$

$$\Delta \mathrm{T}_{Hog} = \gamma \Delta \mathrm{T}_{Filbert} \Rightarrow \mathrm{T}_{Filbert} = \frac{\Delta \mathrm{T}_{Hog}}{\gamma}$$

We just need to find γ, now:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.9\ c)^2}{c^2}}} = \frac{1}{\sqrt{1-0.81}} = \frac{1}{0.43589} = 2.29$$

So, back to the time dilation equation:

$$\mathrm{T}_{Filbert} = \frac{\Delta \mathrm{T}_{Hog}}{\gamma} = \frac{11.1\bar{1}\ years}{2.29} = 4.84\ years$$

(E) Which duck (reference frame) measures the correct values? Well, they all do! That's the weirdest part of all!

(Extra Information) Recall in class last week that we discussed length contraction in terms of this problem. Let's review that. Consider the way we found the time Hog measured for Filbert's flight to Planet Z. We used $time = distance/rate$. So why couldn't we have used that for the solution in **(D)**? Well, we could, only we'd have to use the distance the Filbert measured between the planets. Since Hog measures the proper distance, Filbert measures a smaller distance, which accounts for his measuring a smaller time. Using the length contraction formula, we can show this:

$$L_{Filbert} = \frac{L_{Hog}}{\gamma}$$

$$\Delta \mathrm{T}_{Filbert} = \frac{L_{Filbert}}{v}$$

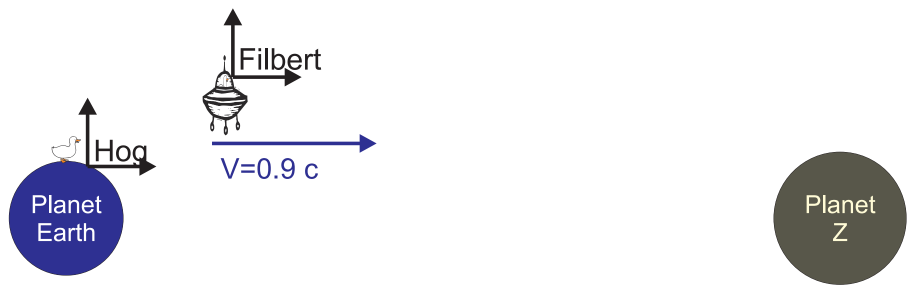
Here v is the speed of Planet Z, according to Filbert, as it rushes toward him. Namely, $v = 0.9\ c$:

$$\Delta \mathrm{T}_{Filbert} = \frac{L_{Filbert}}{0.9\ c} = \frac{1}{\gamma} 10\ light\ years 0.9\ c$$

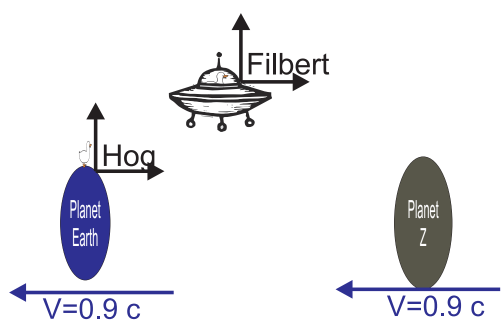
We already calculated what 10 $light\ years/c$ was– that's the time that Hog measured for Filbert's trip– in other words, it's $\Delta \mathrm{T}_{Hog}$. Substituting that in:

$$\Delta \mathrm{T}_{Filbert} = \frac{\Delta \mathrm{T}_{Hog}}{\gamma}$$

This is the time dilation equation (remember Filbert measures the proper time, $\Delta \mathrm{T}_{\circ}$)! So if we wanted to draw more accurate diagrams, we should modify them to take into account this length contraction. This is shown below:



According to Hog, Filbert moves forward at 0.9 c while the Earth and Planet Z sit still.
Hog is also shrunk by a factor of gamma (2.29) along the direction of travel.



According to Filbert, Hog, the Earth and Planet Z move backward at 0.9 c.
Filbert also sees the distance between the planets, their thickness, and Hog's thickness as being shrunk by a factor of gamma (2.29) along the direction of travel.

Now it seems obvious why the travel times are different! The travel distances are different!

7. If you add 100 J of energy to a 1 kg brick, by how much does its mass change?

Answer: To solve this, we use Einstein's most widely-known equation:

$$E = m\,c^2$$

What it says is that just by having mass, something has energy– and a *lot* of it[6]! What it also says is that if you change the energy of something, you change its mass. So if we ass 100 J to a

[6] c^2 is a BIG number!

brick (say, by heating it), we will actually change its mass. Einstein's equation tells us by how much:

$$\Delta E = \Delta m\, c^2$$

$$\Delta m = \frac{\Delta E}{c^2} = \frac{100\ J}{(3 \times 10^8\ m/s)^2} = \frac{10^2\ J}{9 \times 10^{16}\ m^2/s^2} = \frac{1}{9} \times 10^{-14}\ kg = 1.\bar{1} \times 10^{-15}\ kg$$

This is a very small number! Note that instead of recognizing that you could write the equation in terms of changing (Δ) energy leading to changing mass, you could have calculated the energy before adding 100 J, the mass after adding 100 J to that energy, and then subtracting that mass from 1 kg–if your calculator would have handled it!

8. Consider Filbert's spaceship from Problem 6. Assume it has a mass of 10,000 kg.
(A) If Hog were using Newtonian ideas, what would the spaceship's momentum be according to his rest frame?
(B) If Hog were using special relativity, what would the spaceship's momentum be according to his rest frame?

Answer:

(A) Newton says that momentum is equal to $mass \times velocity$. So, according to Newton, Hog would have measured Filbert's momentum to be:

$$P_{Hog} = m \times v = 10,000\ kg \times 0.9\ c = 10,000\ kg \times 0.9 \times 3 \times 10^8\ m/s = 2.7 \times 10^{12}\ kg\, m/s$$

(B) Einstein says that momentum is equal to $\gamma\, mass \times velocity$. So, according Einstein, Hog would have measured Filbert's momentum to be:

$$P_{Hog} = \gamma\, m \times v = 2.29 \times 2.7 \times 10^{12}\ kg\, m/s = 6.183 \times 10^{12}\ kg\, m/s$$