Name: ______________________________ Student I.D.: ______________

Exam 2A

Physics 100, Spring 2007

Friday, March 16, 2007

Useful Equations and Numbers

Acceleration due to gravity...
on Earth = 10 m/s^2
on Mars = 3.7 m/s^2
on the Moon = 1.6 m/s^2

$$\overrightarrow{Impulse} = \overrightarrow{Force} \times Time$$

$$Work = Force_{\parallel} \times Distance$$

$$Work_{net} = \Delta Kinetic\ Energy$$

$$\overrightarrow{Impulse} = \Delta \overrightarrow{Momentum}$$

$$Gravitational\ Potential\ Energy = Mass \times (Acceleration\ of\ Gravity) \times Height$$

$$Kinetic\ Energy = \frac{1}{2}\ mass \times (speed)^2$$

$$\overrightarrow{Momentum} = mass \times \overrightarrow{Velocity}$$

$$\overrightarrow{Force} = mass \times \overrightarrow{Acceleration}$$

$$(Force\ of\ Gravity) = (mass) \times (Acceleration\ due\ to\ gravity)$$

$$a^2 + b^2 = c^2$$

$$displacement = \frac{1}{2} acceleration \times time^2$$

$20\ m/s = 45\ mph$	$1\ mile/minute = 60\ mph$
$1\ pound\ of\ force = 4.5\ Newtons$	$1\ km = 0.6\ miles$
$\sqrt{3^2 + 4^2} = 5$	$1\ hour = 3,600\ seconds$
$1\ Calorie = 4,200\ J$	$1\ g = 10\ m/s^2$
$1\ m/s = 3.6\ km/hour$	$1\ Newton = 1\ \frac{kg\ m}{s^2}$
$1\ m = 3.2\ feet$	$1\ Joule = 1\ kg\ m^2/s^2$
$\sqrt{200} = 14.14$	$1\ Calorie = 1,000\ calories$

DO NOT OPEN EXAM UNTIL INSTRUCTED TO DO SO!
TURN OFF YOUR CELL PHONE!

Answer Key for Exam A

Section 1. **True/False (1 pts. each)**

<u>True</u> The amount of gravitational potential energy something has only depends on its location and not on how it got there.

<u>True</u> If a machine multiplies the force you provide by a factor of *four*, then the amount of distance the object that the machine moves is reduced by a factor of *four*.

<u>False</u> The kinetic energy of a pendulum is at its maximum when it is at the highest point of its swing.

<u>False</u> Momentum is only conserved in *inelastic* collisions.

<u>True</u> A head-on collision between two cars would be more damaging to the occupants if the cars rebounded off of each other than if they stuck together.

<u>True</u> A heavy truck at rest has less momentum than a small fly traveling at 1 millimeter/second.

<u>True</u> Energy is never created or destroyed, only moved around.

<u>False</u> Pushing a boulder up a ramp to an elevation of 3 m imparts it with more potential energy than if you had lifted the boulder straight up 3 m because in pushing it up a ramp you provide a force over a longer distance, doing more work.

<u>True</u> The gravitational potential energy of a pendulum is at its maximum when it is at the highest point of its swing.

<u>False</u> Hydrogen is a *source* of energy that promises to solve the world's energy needs.

<u>True</u> Because the force of friction that stops a car does not depend on the car's speed, the stopping distance for a car moving at 50 km/h is four times that of a car moving at 25 km/h.

<u>False</u> Something can have momentum, but not have kinetic energy.

Section 2. **Multiple Choice (4 pts. each)**

Choose the single best answer unless instructed to do otherwise.

1. You push a physics book across a table at constant speed in a constant direction. Two horizontal forces are acting on the book: the force of friction and the force of your hand. Which of the following are true (circle all that apply)?

   (a) The force of friction is greater in magnitude than the force of your hand.

   [(b)] The force of friction is doing negative work that exactly cancels the positive work that your hand is doing.

   (c) The impulse imparted *by* your hand *on* the book exceeds the impulse imparted *by* friction *on* the book.

   [(d)] The net force acting on the book is zero because it's not changing velocity.

   (e) The net force must be greater than zero because it is moving even though there is friction acting on it.

   (f) The book is accelerating.

   (g) The book has no inertia since it is traveling at constant speed.

   [(h)] The work that friction is doing on the book is transforming the kinetic energy of the book into heat.

   (i) You hate physics books.

2. A 2,000 kg SUV hits a much smaller 1,000 kg compact car which was stopped at a traffic light. Which one vehicle experiences a greater force of impact, and why?

   (a) The SUV, because it was moving fast then changed its momentum more than the compact car while it slowed down.

   (b) The compact car, because it has a smaller inertia and therefore experiences a higher impulse.

   [(c)] They experience the same force because of Newton's 3$^{\text{rd}}$ Law.

   (d) The compact car because it is has much less mass.

3. Jack and his skateboard have a mass of 50 kg and are moving at a speed of 2.5 m/s down a hill. Jill and her skateboard have a mass of 25 kg and are moving with a speed of 5 m/s. Jill has...

   (a) ... twice the momentum as Jack.

   [(b)] ... twice the kinetic energy as Jack.

   (c) ... the same kinetic energy as Jack.

   (d) ... one-half the momentum as Jack.

4. Two lumps of clay with equal and opposite momenta have a head-on collision, stick together, and come to rest. Which of the following is true? (circle all that apply)

   (a) Momentum was conserved.
   (b) Kinetic energy was conserved.
   (c) Total energy was conserved.
   (d) Momentum was not conserved.
   (e) Kinetic energy was not conserved.
   (f) Potential energy was converted into momentum.

5. Norton is lifting a heavy crate at constant speed using a complicated pulley system. Norton notices that after he pulled out 5 meters of rope with a force of 100 N, the crate only went up 0.5 meters. How much does the crate *weigh*?

   (a) 10 N
   (b) 500 N
   (c) 50 N
   (d) 1,000 kg
   (e) 1,000 N

Section 3. **Short Answer Questions (8 pts. each)**

6. A 75 kg skier starts at the top of a 100 m-high hill with a slope of 45 degrees, as shown below in Figure 1. How fast will he be moving when he is at the bottom of the hill just before he hits the ski lodge? Assume he starts from rest and ignore friction and air resistance in your calculations.
Show your work or you won't get full credit!

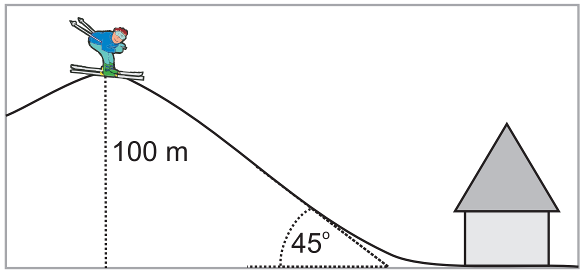


Figure 1: A skier starts from rest at the top of a 100 m high hill. How fast is he moving at the bottom? See Problem 6.

Answer: This is most easily solved using conservation of energy. At the top of the hill, the skier has a total energy of:

$$\begin{aligned} E_{tot,top} &= PE_{top} + KE_{top} \\ &= m\,g\,h + \frac{1}{2}\,m\,v^2 \\ &= (75\ kg)(10\ m/s^2)(100\ m) + 0 \\ &= \mathbf{75,000\ J} \end{aligned}$$

Energy is conserved. Because we are ignoring friction and air resistance, the total mechanical energy the skier has at the bottom of the hill, $E_{tot,bot}$ will be the same as what he had at the top:

$$E_{tot,bot} = PE_{bot} + KE_{bot} = E_{tot,top} = 75,000\ J$$

At the bottom of the hill, the skier has no potential energy, so this equation reduces down to:

$$KE_{bot} = \frac{1}{2}\,m\,v^2 = \frac{1}{2}\,(75\ kg)\,v^2 = 75,000\ J$$

Now, divide both sides of the equation by 75 kg, then multiply by two and you get:

$$v^2 = 2000\ m^2/s^2$$

So that the speed the skier has at the bottom of the hill is:

$$v = \sqrt{2,000}\,m/s = 44.7\ m/s = 100\ mph$$

Note: the world's speed record for skiing is 251.4 km/h = 156 mph set by Simone Origone in Les Arcs France, April 2006. Speed skiers can go this fast because they wear aerodynamic outfits to reduce wind resistance, special skies to reduce friction and plowing in the snow, and ski at high altitudes to further reduce wind resistance. elevations

7. A girl is riding her tricycle. Together they have a total mass of 20 kg and are moving at 5 m/s. How much force must friction provide if she is to stop in a time of 2 seconds so as to avoid hitting her cute, little puppy?
Show your work or you won't get full credit!

Answer: This is an impulse/momentum question. The girl and her tricycle start out with a momentum (magnitude[2]) of:

$$P_{start} = mass \times speed = (20\ kg) \times (5\ m/s) = 100\ kg\,m/s$$

In order to avoid hitting her puppy, she must stop– in other words, she must change her momentum by $-100\ kg\,m/s$. Impulse is defined as:

$$Impulse = \Delta momentum = Force_{\|} \times Time$$

Here the $Force_{\|}$ is friction, and the Time is given as 2 sec. Solving that equation we have:

$$Force_{\|} = \frac{\Delta momentum}{Time} = \frac{-100\ kg\,m/s}{2\ s} = -50\ N$$

And, as usual the answer $(-100\ kg\,m/s)/(2\ s)$ would have been adequate for full credit.

8. In a crash test, a Honda and a BMW hit each other head-on. Originally, the 1,000-kg Honda had a velocity of 25 m/s *north* while the 1,500-kg BMW had a velocity of 20 m/s *south*. After they collide, they stick together.
(A) What direction is the resulting Honda/BMW combination moving?
(B) With what speed is the resulting Honda/BMW combination moving?
Show your work or you won't get full credit!

Answer: **(A)** Total momentum is conserved. So the (vector) sum of the momenta of the two cars before the collision must be equal to the momentum of the combined tangle of the two cars after the collision:

$$\begin{aligned} P_{tot,after} &= P_{tot,b4} \\ &= mass_{Hon}\vec{V}_{Hon} + mass_{BMW}\vec{V}_{BMW} \\ &= (1,000\ kg)(25\ m/s\ north) + (1,500\ kg)(20\ m/s\ south) \\ &= 25,000\ kg\,m/s\ (north) + 30,000\ kg\,m/s\ (south) \\ &= \mathbf{5,000\ kg\,m/s\ (south)} \end{aligned}$$

Since the total momentum of the two cars stuck together points *south*, the Honda/BMW combination must be moving *south*.
(B) We know the momentum of the BMW/Honda combination, and we know it's mass, so the definition of momentum allows us to find the speed[3]:

$$P_{tot,after} = M_{BMW+Honda} \times v = (2,500\ kg) \times v = 5,000\ kg\,m/s$$

$$v = \frac{5,000\ kg\,m/s}{2,500\ kg} = 2\ m/s$$

[2] We are working in one dimension. Remember, in one dimension we can take care of all of the vector aspects of momentum, velocity, acceleration, force, etc. using only the magnitudes and the judicial use of minus signs!

[3] This part of the question just asks for *speed*, so we don't have to consider the direction. That was determined in part **(A)**. Of course, giving the direction doesn't lose any points!

9. Shown in Figure 2 is a boat crossing a river. The river flows to the right at 4 m/s. The boat is cutting directly across the flow of the water. When in still water, the boat moves at 3 m/s.
(A) With what speed will the boat move, relative to the water in the river?
(B) With what speed will the boat move, relative to a baby duck sitting on the shore?
(C) Draw the boat's velocity vector, to scale, according to the baby duck sitting on shore.
Show your work or you won't get full credit!

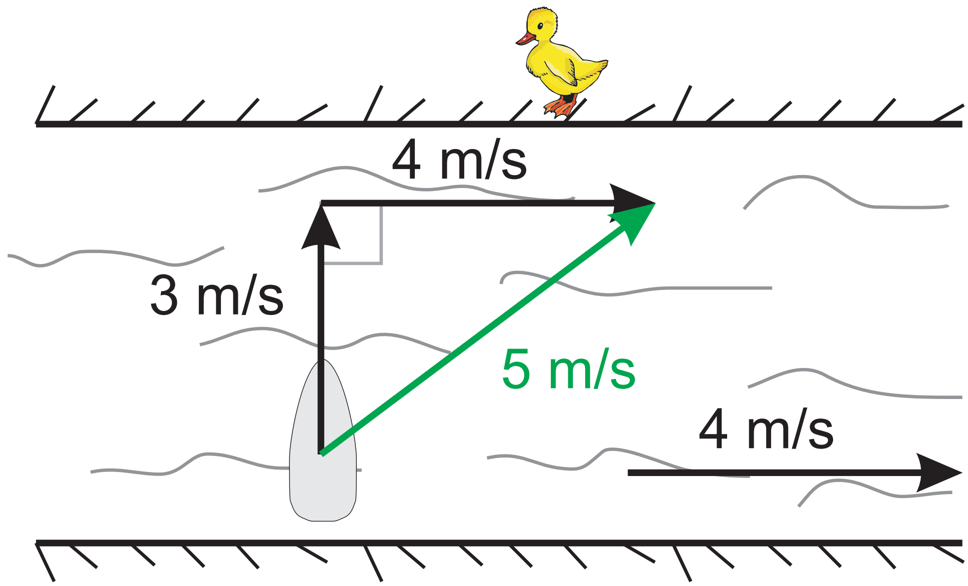


Figure 2: Problem 9: a boat crosses a river.

Answer: **(A)** The boat will move at 3 m/s, relative to the water. Since that is the speed it moves in still water, that is the speed it will mover relative to moving water.

(B) Since the boat moves 3 m/s relative to the water, and the water moves 4 m/s relative to the duck, the motion of the boat relative to the duck is determined by *vectorially* adding the two velocities. The way to do this is either using the parallelogram rule or the head-to-tail method. I prefer the head-to-tail method, and that is shown in the figure: I translated the water's velocity vector up to the boats, placing them head-to-tail. The resultant vector (i.e. their vector sum) then starts at the tail of the boat's velocity vector and ends at the head of the water's velocity vector, as shown. Since the two vectors being added are at right angles to each other, we may use the Pythagorean Theorem to find the resultant's magnitude:

$$V_{tot}^2 = (3\ m/s)^2 + (4\ m/s)^2 = 9\ m^2/s^2 + 16\ m^2/s^2 = 25\ m^2/s^2$$

Take the square root of both sides and you find:

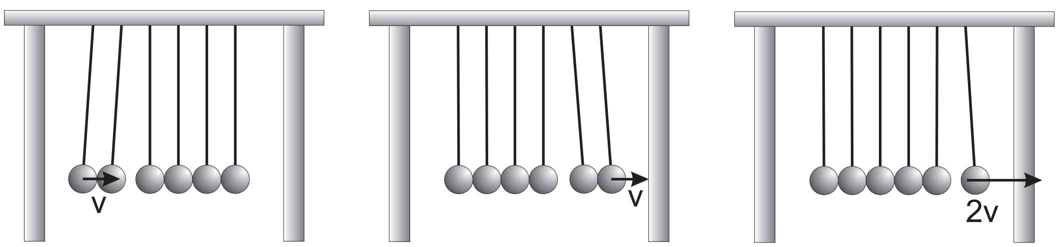
$$V_{tot} = 5\ m/s$$

(C) See Figure 2, above.

10. Consider the classic swinging-balls apparatus, as shown in Figure 3. In case you don't know, a swinging-balls apparatus is made of six identical steel balls suspended in the air by thin strings.
In this problem, to start the balls moving, two of them are lifted and allowed to crash into the other four with a speed of v, as shown Figure 3(a). They collide into the other balls without deformation. Consider two different *cases* that might result from this collision:

I. The case shown in Figure 3(b) where two balls emerge from the other side, each with the same speed, v, as the original impacting balls.

II. The case shown in Figure 3(c) where only one ball emerges, but has a speed of $2v$, twice that of the original, impacting balls.



(a) Two balls impact with speed v

(b) Case **(I)**: Two balls emerge with speed v

(c) Case **(II)**: One ball emerges with speed $2v$

Figure 3: Problem 10: The swinging-balls apparatus.

(A) Do cases **(I)** and **(II)** both conserve momentum? Explain.
(B) Why does the case **(I)** happen in real life while **(II)** never does? Explain why using at *least* one of concepts we've studied in this class. *Hint: ignore friction, air resistance, and sound generation*[4].

Answer: (A) Yes, both cases conserve momentum because the momentum before the collision is equal to the momentum after the collision for both cases. To show this: In terms of the mass of a single steel ball, the original momentum is:

$$P_\circ = mass \times velocity = 2\,m\,v \ (towards\ right)$$

For case **(I)**, the resulting momentum would be:

$$P_{\mathbf{(I)}} = mass \times velocity = (2\,m) \times (v\ (towards\ right)) = P_\circ$$

Therefore, momentum is conserved for case **(I)**.
For case **(II)**, the resulting momentum would be:

$$P_{\mathbf{(II)}} = mass \times velocity = (m) \times (2\,v\ (towards\ right)) = P_\circ$$

Therefore, momentum is also conserved for case **(II)**.

(B) The reason that case **(II)** never happens in real life is that it doesn't conserve mechanical energy. In class, we discussed how elastic collisions conserve mechanical

[4]This hint was announced at the very beginning of the exam and written on the board.

energy, while elastic collisions do not[5]. Even if you did not attend that lecture, we have discussed how mechanical energy is conserved if there isn't anything to take it away, like friction, the generation of sound, or the deformation of things.

Since the balls are all at the same height just before/after the collisions, their potential energy is not a factor in the conservation of mechanical energy. Therefore, in this case, kinetic energy must be conserved.

For case **(I)**, kinetic energy is clearly conserved because before the collision, there are two balls moving at the speed v, and after the collision there are two balls moving at the speed v– it's basically the same situation.

That is not true for case **(II)**: the kinetic energy before is:

$$KE_\circ = \frac{1}{2}\,(2m) \times (v)^2 = \,m\,v^2$$

While the kinetic energy would be:

$$KE_{\mathbf{(II)}} = \frac{1}{2}\,(m) \times (2\,v)^2 = 2\,m\,v^2$$

So, for case **(II)** to happen, there would have to be some additional mechanical energy added to the system, and there is no mechanism for that to happen. Therefore case **(II)** violates energy conservation.

[5] For those of you that were there, recall the air carts discussion of elastic/inelastic collisions and energy conservation.