5 Subtraction

5.1 Subtracting Money

Activity 5.1 Money Activity: Borrowing from the Bank

Objective: To solidify concepts of subtraction by using money.

Materials: Bills of different denominations.

Group size: 4.

In your groups of four, select a person to start as the banker (group member #1) and select who will be member #2, member #3, and member #4. Using denominations $10,000, $1000, $100, $10, $1, dimes and pennies, the banker will hand out the original amount of money using the fewest number of bills/coins possible. Then, when needed, exchange bills so that the loan can be made.

Write the names of each group member:

Group member #1 ______________________
Group member #2 ______________________
Group member #3 ______________________
Group member #4 ______________________

1. Group member #1 is the banker. Begin by counting out $347.10 and give it to member #2. Make sure that you take the *least* number of each denomination that makes up this amount. Record how many of each denomination member #2 has:

$\$10,000$	$\$1000$	$\$100$	$\$10$	$\$1$	dimes	pennies

   (a) Group member #3 takes $20 from member #2.

      i. Can member #2 give the money to member #3 without exchanging bills with the bank?

      ii. If not, why not? What will they have to exchange to make the gift possible?

      iii. How much does member #2 have left after the gift? Record how many of each denomination member #2 has:

$\$10,000$	$\$1000$	$\$100$	$\$10$	$\$1$	dimes	pennies

      iv. Write a subtraction equation that models this event.

(b) Now, group member #4 takes $50 from what member #2 has left.

i. Can member #2 give the money without exchanging bills with the bank?
ii. If not, why not? What will they have to exchange to make the loan possible?
iii. How much does member #2 have left after the gift? Record how many of each denomination member #2 has:

$\$10,000$	$\$1000$	$\$100$	$\$10$	$\$1$	dimes	pennies

iv. Write a subtraction equation that models this event.

2. Now, start over giving all the money back to the bank. The banker counts out $1258.92 to group member #3. Make sure that you take the *least* number of each denomination that makes up this amount. Record how many of each denomination member #3 has:

$\$10,000$	$\$1000$	$\$100$	$\$10$	$\$1$	dimes	pennies

(a) Group member #2 takes $225 from member #3.

i. Can member #3 lend the money without exchanging bills with the bank?
ii. If not, why not? What will they have to exchange to make the gift possible?
iii. How much does member #3 have left after the gift? Record how many of each denomination member #3 has:

$\$10,000$	$\$1000$	$\$100$	$\$10$	$\$1$	dimes	pennies

iv. Write a subtraction equation that models this event.

(b) Now, group member #4 takes $50 from what member #3 has left.

i. Can member #3 give the money without exchanging bills with the bank?
ii. If not, why not? What will they have to exchange to make the gift possible?
iii. How much does member #3 have left after the gift? Record how many of each denomination member #3 has:

$\$10,000$	$\$1000$	$\$100$	$\$10$	$\$1$	dimes	pennies

iv. Write a subtraction equation that models this event.

3. Compare your results with another group in the class.
4. What strategy could you use to estimate the answers? How would this compare with the methods used to estimate sums?
5. How would you compare this activity with the method for subtraction that you know?

5.2 The process of subtraction:

Please notice that every time you were lending money during the previous activity, if the number of bills from any denomination was larger than the bank's number of bills from the same denomination, the bank had to make an exchange for equal amount of money with higher denomination bills. For example, assume bank had $25 as two $10 bills and five $1 bills and you were borrowing $8. The bank had to exchange one of its $10 bills with ten $1 bills to make its $25 to be as one $10 bill and fifteen $1 bills. Now, to borrow eight $1 bills the bank has enough $1's. The banks balance would be $17, as one $10 bill and seven $1 bills.

We use the same idea of exchanging when we subtract any two numbers.

Vocabulary

- The operation is called **subtraction**.
- The number to subtract from is called the **minuend**.
- The number being subtracted is called the **subtrahend**.
- The result of the operation is called the **difference**.

For example, in $8 - 5 = 3$, the 8 is the minuend, the 5 is the subtrahend, and the 3 is the difference.

Steps for subtracting whole numbers and decimals:

1. As for addition, line up the numbers, digit by digit, so that each place value is in the same column. The number you are subtracting from (Minuend) should be on top, and the number you are subtracting (Subtrahend), below that.
2. Start from the right most column.
3. If the digit on top is greater than or equal to the digit on the bottom, take away the digit on the bottom from the digit on the top and write the difference below the bottom digit in the same column.
4. If the digit on top is smaller than the digit on the bottom, exchange from the digit to the left on top, giving you an extra ten, enough to take away. To indicate the extra ten, write a small 1 just to the left of the top number's digit. Then take away the bottom and write the difference below the bottom digit.

Example 1: $7,458.39 - 5,297.58$

Solution:

$$\begin{array}{rcccccccc} & 7 & , & 4 & 5 & 8 & . & 3 & 9 \\ - & 5 & , & 2 & 9 & 7 & . & 5 & 8 \\ \hline & & & & & & & & 1 \end{array} \longrightarrow \begin{array}{rcccccccc} & & & & & 7 & & & \\ & 7 & , & 4 & 5 & \cancel{8} & . & {}^{1}3 & 9 \\ - & 5 & , & 2 & 9 & 7 & . & 5 & 8 \\ \hline & & & & & & & 8 & 1 \end{array}$$

$$\begin{array}{rcccccccc} & & & 3 & & 7 & & & \\ & 7 & , & \cancel{4} & {}^{1}5 & \cancel{8} & . & {}^{1}3 & 9 \\ - & 5 & , & 2 & 9 & 7 & . & 5 & 8 \\ \hline & & & & & 0 & . & 8 & 1 \end{array} \longrightarrow \begin{array}{rcccccccc} & & & 3 & & 7 & & & \\ & 7 & , & \cancel{4} & {}^{1}5 & \cancel{8} & . & {}^{1}3 & 9 \\ - & 5 & , & 2 & 9 & 7 & . & 5 & 8 \\ \hline & 2 & , & 1 & 6 & 0 & . & 8 & 1 \end{array}$$

So, the difference of 7,458.39 and 5,297.58 is 2,160.81. Granted, with all of the markings for the exchanging, the process can get pretty messy.

- If your handwriting is messy already, try writing bigger.
- Everyone needs to be careful to keep the place value columns in line.

One nice advantage to subtraction, is that you can check your answer with addition. If your difference is correct, then the difference plus the subtrahend will equal the minuend. In our case, to check, we see that $2,160.81 + 5,297.58 = 7,458.39$

$$\begin{array}{rcccccccc} & & & 1 & & 1 & & & \\ & 2 & , & 1 & 6 & 0 & . & 8 & 1 \\ + & 5 & , & 2 & 9 & 7 & . & 5 & 8 \\ \hline & 7 & , & 4 & 5 & 8 & . & 3 & 9 \end{array}$$

Example 2: $24,215.4-35.17$

Solution:
Notice the extra zero in the minuend as a place-holder for the hundredths place.

$$\begin{array}{rcccccccc} & & & & & & & 3 & \\ & 2 & 4 & , & 2 & 1 & 5 & . & \not{4} & {}^{1}0 \\ - & & & & & 3 & 5 & . & 1 & 7 \\ \hline & & & & & & & & & 3 \end{array} \longrightarrow \begin{array}{rcccccccc} & & & & 1 & & & & 3 & \\ & 2 & 4 & , & \not{2} & {}^{1}1 & 5 & . & \not{4} & {}^{1}0 \\ - & & & & & 3 & 5 & . & 1 & 7 \\ \hline & & & & & & 0 & . & 2 & 3 \end{array}$$

$$\begin{array}{rccccccccc} & & & & 1 & & & & 3 & \\ & 2 & 4 & , & \not{2} & {}^{1}1 & 5 & . & \not{4} & {}^{1}0 \\ - & & & & & 3 & 5 & . & 1 & 7 \\ \hline & 2 & 4 & , & 1 & 8 & 0 & . & 2 & 3 \end{array}$$

So, the difference of 24,215.4 and 35.17 is 24,180.23. Use addition to check:

$$\begin{array}{rccccccccc} & & & & 1 & & & & 1 & \\ & 2 & 4 & , & 1 & 8 & 0 & . & 2 & 3 \\ + & & & & & 3 & 5 & . & 1 & 7 \\ \hline & 2 & 4 & , & 2 & 1 & 5 & . & 4 & 0 \end{array}$$

Exercise 5.2.1 Find the following differences to practice subtraction. Add back to check.

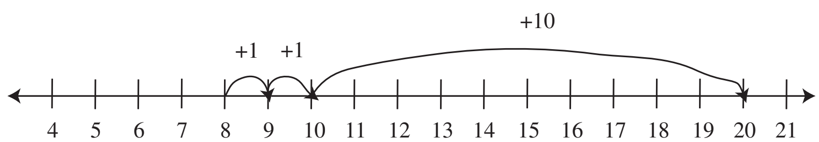
1. $236,972.112-35,387.17$
2. $76,002.17-12,378.18$
3. $38,937.08-12,609.5$
4. $237.16-59.98$

5.3 Alternate subtraction method: Subtraction by Counting Up

If you are working retail and have to give change for a purchase, you can use the method of counting up. In fact, to be polite and let the customer know that you are doing it correctly, you would count up out loud!

Example 1: Suppose a customer pays for an \$8 item with a \$20. How would you give change?

Solution: As you counted back the proper bills you might say something like, "\$8, 9, 10, and 10 is 20", as you hand the customer two ones and a ten. On a number line, this counting up procedure looks like:



The total change is the total of the jumps, that is, \$12.

When we subtract using the standard method introduced in the previous section, we get a result that looks something like the example below.

Example 2:

$$\begin{array}{rr} 5 & 4 \\ - \; 2 & 9 \\ \hline \end{array} \quad \longrightarrow \quad \begin{array}{rr} {}^{4}\not{5} & {}^{1}4 \\ - \; 2 & 9 \\ \hline 2 & 5 \end{array}$$

There are two important things to notice about the result:

(1) We can check our work by adding the bottom two numbers to see if we get the top number (does $25 + 29 = 54$?).

(2) This gives us an alternative way of subtracting numbers. Since the result of subtraction (the difference) adds with the lower number (the subtrahend) to make the top number, we can figure out the difference by counting up from the bottom to the top. For example, let's use this approach with the example from above:

$$\begin{array}{rr} 5 & 4 \\ - \; 2 & 9 \\ \hline \end{array} \xrightarrow{+1} 30 \xrightarrow{+20} 50 \xrightarrow{+4} 54$$

By adding 1 to 29 we jump to 30, a nice round number, and a relatively easy number to jump to 50 from - just add 20. Once we're at 50 we only have to add 4 more to get to 54. If we add up the numbers we used to make our jumps: $1 + 20 + 4 = 25$, we get the difference.

Try the next example on your own before reading the solution:

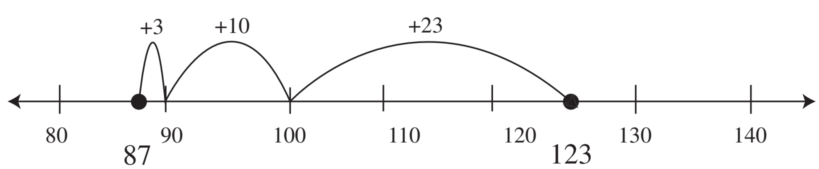
Example 3:

$$\begin{array}{r} 1\ 2\ 3 \\ -\ \ 8\ 7 \\ \hline \end{array}$$

Solution:

$\begin{array}{r} 1\ 2\ 3 \\ -\ \ 8\ 7 \\ \hline \end{array} \xrightarrow{+3} 90 \xrightarrow{+10} 100 \xrightarrow{+23} 123$ Adding the jumps gives us $3 + 10 + 23 = 36$ so we know that $123 - 87 = 36$.

The graph below shows what these jumps look like on a number line.



These examples offer one possible way to count up from one number to another. You might have chosen different jumps than those shown here and so long as you add your jumps correctly when you count them up, the jumps you choose are up to you.

Exercise 5.3.1 Continue finding differences to practice subtraction. Include appropriate units in your answers.

Find these differences by counting up.

1. $123 - 88$
2. $275 - 189$
3. $204 - 178$
4. $1782 - 945$
5. $1.58 - 0.71$
6. How much change would you get on a charge of $8.39 if you pay with a $20?
7. How much change would you get on a charge of $17.62 if you pay with a $20?
8. How much change would you get on a charge of $33.14 if you pay with a $50?
9. How much change would you get on a charge of $47.23 if you pay with a $100?

Find these differences using any method. Show your work clearly, so that someone looking at it could tell what method you are using. Then, check your result with a calculator.

10. $4,398.9 - 51.71$

11. $192,543.09 - 76,119.12$

12. $321,311.61 - 99,378.5$

13. $3,012.9 - 2,981.16$

14. $6,429,199 - 399.05$

15. $12,001,260.01 - 10,652,981.7$

Find The Patterns

Exercise 5.3.2 For the following lists of numbers,

- Fill in the missing numbers.
- Describe the pattern in words. (The first one is done for you.)

1. $2, 4, 6, 8, \_\_\_\_, 12, \ldots$ Pattern: The numbers go up by two each time.

2. $10, 13, 16, \_\_\_\_, 22, 25, \ldots$ Pattern:

3. $28, 23, \_\_\_\_, \_\_\_\_, 8, 3$ Pattern:

4. $\frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \_\_\_\_, 1\frac{3}{8}, 1\frac{5}{8}, \_\_\_\_ \ldots$ Pattern:

5. $32.7, 32.3, \_\_\_\_, \_\_\_\_, 31.1, \ldots$ Pattern:

6. $23, \_\_\_\_, 35, \_\_\_\_, 47, \_\_\_\_, 59, \ldots$ Pattern:

7. $\frac{1}{2}, 1\frac{1}{4}, \_\_\_\_, 2\frac{3}{4}, \_\_\_\_, \ldots$ Pattern:

8. $14.8, \_\_\_\_, 18, \_\_\_\_, \_\_\_\_, 22.8, \ldots$ Pattern:

9. $14\frac{1}{2}, 11, \_\_\_\_, 4, \_\_\_\_$ Pattern:

Find The Digits

Exercise 5.3.3 Fill in the blanks to complete the addition and subtraction problems below.

1.

$$\begin{array}{cccc} & 5 & 1 & 8 \\ + & 3 & \square & 3 \\ \hline & \square & 6 & 1 \end{array}$$

2.

$$\begin{array}{cccc} & 1 & 7 & 3 \\ + & & \square & 4 \\ \hline & 2 & 6 & 7 \end{array}$$

3.

$$\begin{array}{ccccc} & & \square & 2 & 7 \\ + & & 5 & 9 & \square \\ \hline & 1 & 5 & \square & 1 \end{array}$$

4.

$$\begin{array}{ccccc} & 1 & 2 & \square & 2 \\ - & & 4 & 3 & 3 \\ \hline & & 8 & 5 & \square \end{array}$$

5.

$$\begin{array}{cccc} & 1 & \square & 6 \\ - & & 5 & \square \\ \hline & \square & 3 & 4 \end{array}$$

6.

$$\begin{array}{ccccc} & 2 & 5 & 3 & 2 \\ - & \square & 8 & 1 & \square \\ \hline & & \square & 1 & 6 \end{array}$$

7.

$$\begin{array}{ccccc} & & 6 & 5 & \square \\ + & & 8 & \square & 5 \\ \hline & 1 & 5 & 0 & 8 \end{array}$$

8.

$$\begin{array}{cccc} & 6 & 4 & \square \\ - & \square & \square & 8 \\ \hline & 3 & 4 & 9 \end{array}$$

9.

$$\begin{array}{ccccc} & \square & 0 & 3 & 5 \\ - & & 6 & 3 & \square \\ \hline & & 3 & \square & 8 \end{array}$$

10.

$$\begin{array}{cccc} & 5 & 7 & \square \\ & & \square & 6 \\ + & 2 & 4 & 3 \\ \hline & \square & 9 & 6 \end{array}$$

11.

$$\begin{array}{ccccc} & & 8 & \square & 2 \\ & & \square & 4 & 6 \\ + & & & & \square \\ \hline & \square & 3 & 8 & 4 \end{array}$$

12.

$$\begin{array}{ccccc} & \square & \square & 5 & 3 \\ & & 5 & 3 & 7 \\ + & & 6 & \square & 2 \\ \hline & 4 & 8 & 9 & \square \end{array}$$

5.4 Estimation or Approximation

An Estimation Technique for Subtraction:

- Note the largest place value in the largest number.
- Round each number to the place value noted.
- Subtract the rounded numbers.

Example 1: Estimate the difference:

$7,649 - 3,452$

Solution:

7,649 rounded to the nearest thousand is 8,000.

3,452 rounded to the nearest thousand is 3,000.

The difference of 8,000 and 3,000 is 5,000.

The estimate of the difference of 7,649 and 3,452 is 5,000.

Example 2: Estimate the difference:

$159.99 - 20.898$

Solution 1:

159.99 rounded to the nearest hundred is 200.

20.898 rounded to the nearest hundred is 0.

The difference of 200 and 0 is 200.

The estimate of the difference of 159.99 and 20.898 is 200.

Notice that although this estimate is easy to make, the value is far from the actual. An alternative is to round to the largest place value of the smaller number. This makes the problem a little more acurate and usually only a little more difficult.

Solution 2: The smallest number in the difference is 20.898 and its largest place value is the ten's place. Therefore, we will round both numbers to the nearest 10:

20.898 rounded to the nearest 10 is 20.

159.99 rounded to the nearest 10 is 160.

The difference of 160 and 20 is 140.

A more accurate estimate of the difference of 159.99 and 20.898 is 140.

Exercise 5.4.1 Approximate Difference Practice

For each of the following differences:

(a) Estimate the difference by rounding each number before subtracting.
(b) Find the actual difference.
(c) State whether the actual difference is $>$, $=$, or $<$ the estimate.

1. $45,467.9 - 128.4$
2. $890.13 - 632.9$
3. $70.5 - 19.12$
4. $3,749,221.11 - 67,921.5$
5. $2,039 - 1934.5$
6. $11,012 - 9,261$
7. $12,135,295 - 2,367,109$
8. $17,726.14 - 17,632$
9. $10,138.22 - 8,924$
10. $1.56 - 0.84$

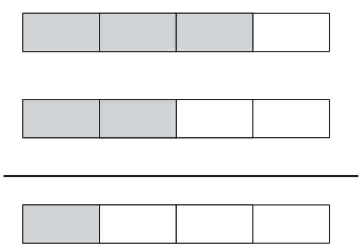
5.5 Subtracting Fractions with Like Denominators

The idea of subtracting fractions is the same as with addition. For example, if you have three quarters and take away two quarters, you have one quarter left. Back in the early days that would look like:

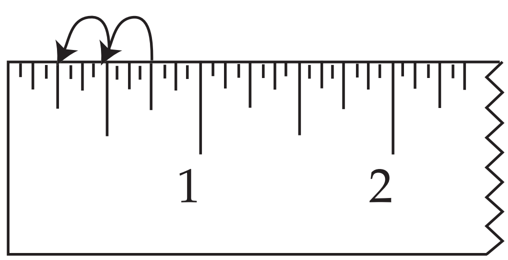
$$\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$$

With rectangles representing each whole, a representation of this problem might look like:

$$\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$$



Or, on a ruler if you start at $\frac{3}{4}''$ and jump back $\frac{2}{4}''$, you will land at $\frac{1}{4}''$.



Notice that just like with addition, the denominators *must* be the same in order for the problem to be possible.

Just as important, when we represent the subtraction problem with a picture, we *must* remember to make the rectangles representing each whole the same size and shape, and then make sure the fractional pieces within each whole are the same size and shape as well.

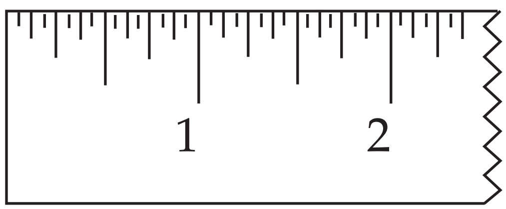
Exercise 5.5.1 For each of the following differences:

(a) Draw a picture using shaded rectangles representing the difference.
(b) Represent the difference on the given ruler.
(c) Find the result of the difference.

1. $\frac{3}{4} - \frac{1}{4}$

   (a) Drawing with rectangles:

   (b) Representation on a ruler:

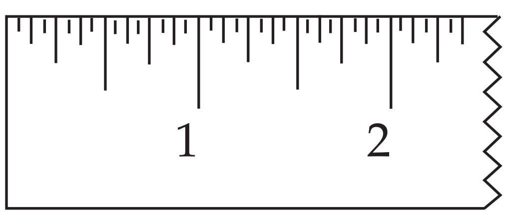


   (c) The result:

2. $\frac{5}{8} - \frac{3}{8}$

   (a) Drawing with rectangles:

   (b) Representation on a ruler:

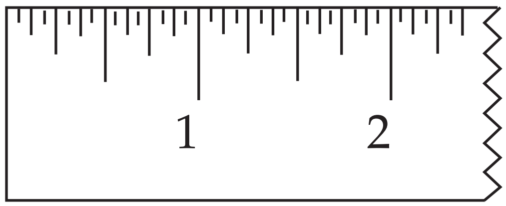


   (c) The result:

3. $\frac{3}{2} - \frac{1}{2}$

(a) Drawing with rectangles:

(b) Representation on a ruler:

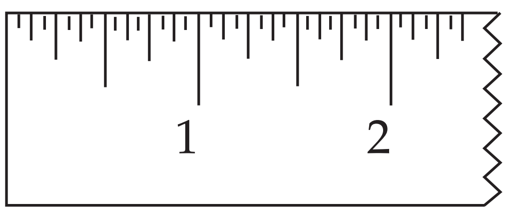


(c) The result:

4. $\frac{5}{4} - \frac{3}{4}$

(a) Drawing with rectangles:

(b) Representation on a ruler:

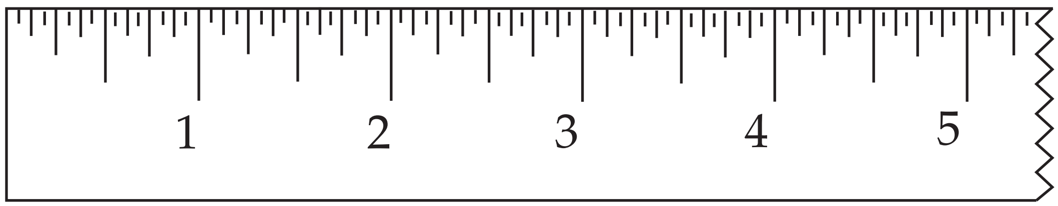


(c) The result:

5. $\frac{7}{2} - \frac{1}{2}$

   (a) Drawing with rectangles:

   (b) Representation on a ruler:

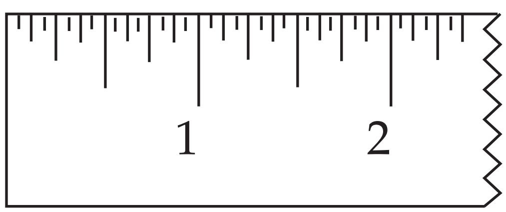


   (c) The result:

6. $\frac{7}{4} - \frac{1}{2}$

   (a) Drawing with rectangles:

   (b) Representation on a ruler:

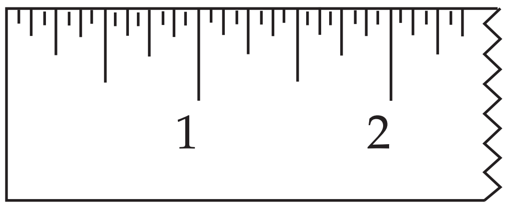


   (c) The result:

7. $\frac{7}{8} - \frac{1}{2}$

(a) Drawing with rectangles:

(b) Representation on a ruler:



(c) The result:

Some of the answers in the last exercise could be written as an *equivalent fraction* with a smaller denominator. For the problems in which this is true, check "yes", and write the original difference as well as the simpler, equivalent fraction. If there is no simpler, equivalent fraction, check "no". The second one is done for you.

Problem	Yes	No	Original Fraction	Equivalent Fraction
1				
2	√		$\frac{2}{8}$	$\frac{1}{4}$
3				
4				
5				
6				
7				

5.6 Comparing Fractions

How can we decide which fraction is bigger?

- If the fractions have the same denominator, then the size of the pieces is the same, and we are just comparing the number of pieces.

Example 1:
We can compare $\frac{3}{4}$ and $\frac{2}{4}$ by seeing that $\frac{3}{4}$ has one more fourth than $\frac{2}{4}$ does:

$$\frac{3}{4} \quad > \quad \frac{2}{4}$$

- If the fractions **do not** have the same denominators, we need to find equivalent fractions for each that **do** have the same denominator:

Example 2:
How can we compare $\frac{5}{8}$ and $\frac{1}{2}$? By seeing that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$, we can see that $\frac{5}{8}$ is bigger:

$$\frac{5}{8} \quad ? \quad \frac{1}{2}$$

ahhh...

$$\frac{5}{8} \quad > \quad \frac{4}{8}$$

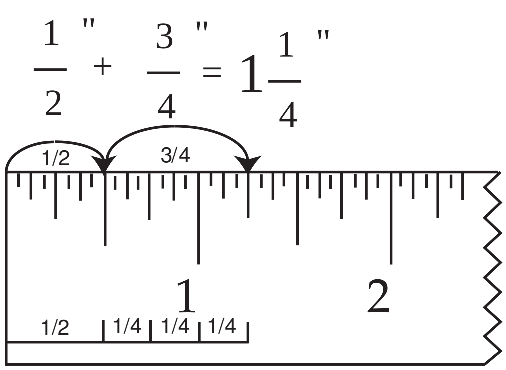
Exercise 5.6.1 For the following pairs of fractions,

(a) Draw a picture for each to figure out which is bigger. (b) Place a $>$, $=$, or $<$ symbol between the fractions to indicate their relative size.

1. $\frac{3}{4}$ and $\frac{1}{4}$
2. $\frac{5}{8}$ and $\frac{7}{8}$
3. $1\frac{1}{3}$ and $\frac{2}{3}$
4. $\frac{3}{4}$ and $\frac{3}{2}$
5. $\frac{1}{2}$ and $\frac{3}{8}$
6. $\frac{1}{4}$ and $\frac{3}{8}$
7. $\frac{3}{4}$ and $\frac{5}{8}$
8. $1\frac{5}{8}$ and $1\frac{3}{4}$

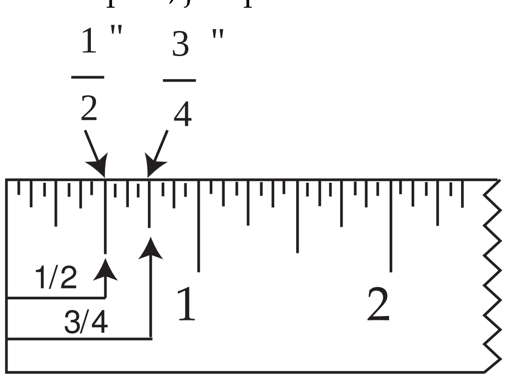
Recall: Adding Fractions on a Ruler

Another way to visualize which fraction is bigger is to think of them as lengths. Recall that when we were using the rulers to visualize the addition of fractions, we identified where the first fraction sat on the ruler, and then added the second fraction by jumping ahead a length equivalent to the other fraction. For example, to illustrate $\frac{1}{2} + \frac{3}{4}$, we drew a picture similar to the following:



Comparing Fractions on a Ruler

To compare, just place each fraction on the ruler, and see which one is a longer length!

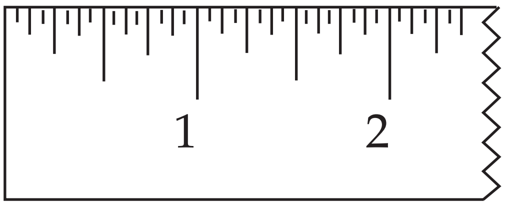


Exercise 5.6.2

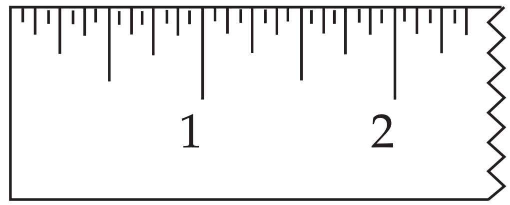
For the following pairs of fractions,

- Indicate where each fraction lies on the ruler to see which is bigger.
- Place a $>$, $=$, or $<$ symbol between the fractions to indicate their relative size.

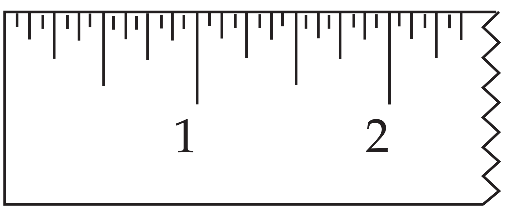
1. $1\frac{1}{8}$ and $1\frac{1}{4}$



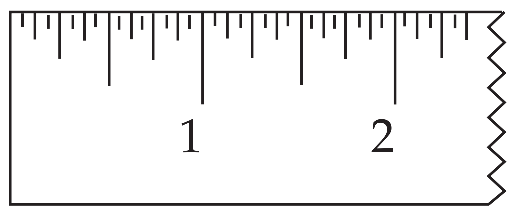
2. $\frac{3}{4}$ and $\frac{3}{8}$



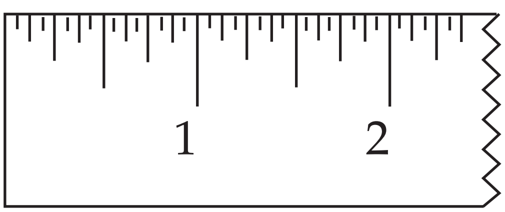
3. $\frac{5}{16}$ and $\frac{3}{8}$



4. $\frac{3}{4}$ and $\frac{7}{8}$



5. $\frac{1}{2}$ and $\frac{7}{16}$



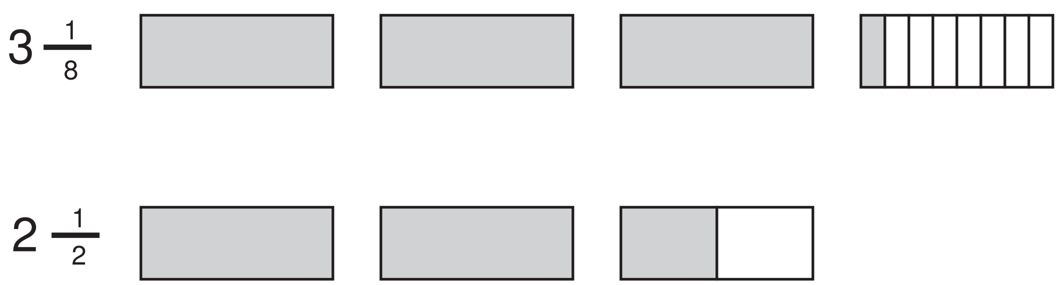
5.7 Subtraction in Real Life

One of the first places you will encounter subtraction after this class is...in your next math class! The most common words that are used to imply subtraction in math problems are "difference between" and "subtracted from". Because the order that numbers are subtracted changes the difference (unlike addition in which order doesn't matter), it is important to know what order is implied.

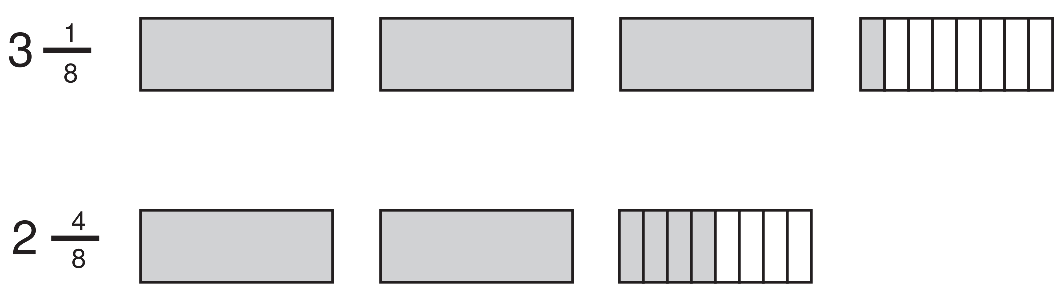
Example 1: What is the difference between $3\frac{1}{8}$ and $2\frac{1}{2}$?

Solution:

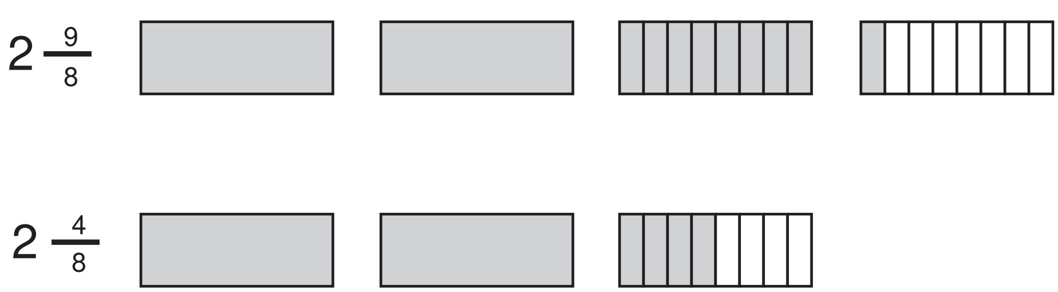
We can find the difference by subtracting $3\frac{1}{8} - 2\frac{1}{2}$:



From previous problems and pictures, we have seen that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$. We therefore break up the $\frac{1}{2}$ in $2\frac{1}{2}$:



So, our difference is equivalent to $3\frac{1}{8} - 2\frac{4}{8}$. Because 4 eighths is more than 1 eighth, we have to borrow 8 eighths from the third whole. Now $3\frac{1}{8}$ is equivalent to $2\frac{9}{8}$:



Without the pictures, this process looks like:

$$
\begin{array}{r}
\overset{2}{\not{3}}\ \frac{1}{8} \quad \curvearrowright \frac{8}{8} \\
-\ 2\ \frac{4}{8} \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
\overset{2}{\not{3}}\ \frac{9}{8} \\
-\ 2\ \frac{4}{8} \\
\hline
\frac{5}{8}
\end{array}
$$

So, the difference between $3\frac{1}{8}$ and $2\frac{1}{2}$ is $\frac{5}{8}$.

Example 2: What is 179.95 subtracted from 438.50?

Solution:

When the words “subtracted from” are used, the order that the numbers appear is opposite the order that they are subtracted. 179.95 subtracted from 438.50 is equivalent to the difference between 438.50 and 179.95 which is 258.55. (Verify this! Math books often leave out details in an example if it’s something that they think you have already done. For practice and to make sure, you should always go through on your own any details that are left out.)

So, 179.95 subtracted from 438.50 is 258.55.

In a more real life setting, there are three common situations where subtraction is used. They are finding a **difference** in two quantities, finding **how much more** you need, and finding the **change** in a quantity.

Difference: If you want to know how far apart two quatities are, literally find the **difference** in height, weight, price, bank balance, or anything that has size, subtract the smaller quatity from the larger quantity.

Example 1: At their 5 year check-up, the doctor weighed each twin. The older twin weighed 42.3 lbs, while the younger twin weighed 33.5 lbs. How much heavier is the older twin than the younger twin?

Solution: It is common in math problems, that the idea is the same as something you've studied, but the words used to describe the idea are different. You must practice the process of understanding the given words, by reading the passage many times and thinking clearly about what it *means*, then thinking of what familiar math idea that it is equivalent to. Only through practice, and asking questions if a certain problem doesn't make sense, will you get good at this. In this case, although the word "difference" is not used in the problem, the given question is equivalent to the question, "What is the difference in their weights?". The difference in their weights is:

$$42.3 - 33.5 = 8.8$$

So, the older twin is 8.8 lbs heavier than the younger twin.

How Much More: If you have a certain amount of something and want to know **how much more** you need to get to a total, subtract how much you have from the total that you want. The difference is how much more you need.

Example 2: A math teacher is saving to buy a 2002 Prius 4-door which costs \$15,998.95. He has already saved \$6,362.17. How much more does he need to save?

Solution: This time the wording in the problem is identical to our category. The required amount to save is how much he has saved already, \$6,362.17, subtracted from the total that he needs for the car \$15,998.95. :

$$15,998.95 - 6,362.17 = 9,636.78$$

So, he needs to save \$9,636.78 more.

Change: The final common use for subtraction that we will discuss here, is finding the **change** in a quantity. Whether something gets bigger or smaller, the change is the bigger value minus the smaller value.

Example 3: The percentage of young adults (age 18 to 24 years) who voted in various presidential elections are shown in the table:

Year	Percent
1972	50
1980	40
1984	41
1988	36
1996	32
2000	26

What was the change in the percentage of young adults voting between the years 1980 and 2000?

Solution: Here is a time where we use the identification numbers 1980 and 2000, to look up the values we need for the calculation. The percentage changed from 40 to 26 during that time, so the change is:

$$40 - 26 = 14.$$

So, the percentage changed by 14%.

Exercise 5.7.1 Answer the following problems. Use addition or subtraction as appropriate. Include appropriate units in your answers.

Difference Between:

1. Find the difference between $5\frac{3}{8}$ and $4\frac{3}{4}$.
2. Find the difference between $12\frac{1}{2}$ and $7\frac{7}{8}$.
3. Find the difference between $\frac{7}{10}$ and 0.23.

4. Find the difference between $\frac{23}{100}$ and 0.09.

Subtracted From:

5. What is 14,375.04 subtracted from 32,191.23?

6. What is 17.8% subtracted from 55%?

How Much More:

7. On Tuesday, the high temperature in Pacifica was 58.7°F. By Saturday, the high temperature was 64.9°F. How much hotter was it on Saturday than on Tuesday?

8. In a math class, a student needs at least 1250 points to earn an A grade. Letty already has 987 points. How many more points does she need in order to earn her A?

9. Will was 4 feet 3 inches tall. His brother Billy was 3 feet 10 inches tall. How much taller was Will than Billy?

Change:

10. In 1993 the number of deaths in the U.S. due to the AIDS virus was 45,381. In 1999, the number of AIDS deaths in the U.S. was down to 16,273. What was the change in number of AIDS deaths?

11. In San Mateo County, during the 2005-2006 school year, 32.3% of 5th graders achieved 6 out of 6 fitness standards. The next year, 36.6% of 5th graders achieved 6 out of 6 fitness standards. By what percent did it change?

12. The life expectancy of a person born in the U.S. was 73.7 years in 1980, and 77.0 years in the year 2000. What was the change in life expectancy over that time?

Mixed Problems:

13. During the Iowa caucuses in 2008, Obama picked up 38% of the democratic delegates, Edwards got 30%, and Clinton got 29%. What percent was left over for the other candidates?

14. Jing-Jing was putting up a shelf in her closet. She needed a 2-by-4 that was $54\frac{5}{8}$ inches long, but the lumber yard sold her a 2-by-4 that was 8 feet long. How much does she have to cut off to make the board the correct length?

15. In preparing dinner for the guests, Billy needed $2\frac{1}{2}$ cups of chicken broth for the soup, and another $\frac{3}{4}$ cups of chicken broth for the pasta sauce. How much chicken broth does he need to prepare the dinner?

16. Paul was working the concessions stand at the basketball game. A customer ordered a popcorn for $3.50, a hotdog for $4.25, two sodas for $0.90 each, and a candy bar for $0.85. The customer gave Paul a 20 dollar bill. How much change does Paul have to give the customer?

17. If you are planning to attach a board that is $\frac{5}{8}''$ onto a board that is $\frac{3}{4}''$ thick,

    (a) What is the longest screw that you can use?

    (b) What is the shortest screw you can use?

    (c) What is a reasonable range of screw lengths?

18. Find the difference between $22\frac{1}{8}$ and $18\frac{1}{2}$.

19. The balance in Hector's checking account was $134.72 before the deposit. After the deposit, the balance was $473.28. How much was the deposit?

20. Geoff saws $13\frac{7}{8}$ inches off of a branch that measured $20\frac{3}{16}$ inches. How long is the branch after Geoff makes the cut?

21. Find the difference between 48% and 29%.

22. Garret wanted to try to bake bread. The recipe called for $4\frac{1}{2}$ cups of flour, but when he looked in the cupboard, he saw that he only had $2\frac{1}{8}$ cups left. How much more flour does he need?

5.8 Negative Numbers

Activity 5.2 Debt

Objective: To introduce concepts of negative numbers by using money and debt.

Materials: Bills of different denominations. Green bills for "positive" money, and red bills for debt.

Group size: 2.

Section A

Imagine you have a bank account. For the following, count out green bills to illustrate the ideas:

1. If you make an initial deposit of $23, then withdraw $18, how much do you have left in your balance?
2. Write this scenario out as a subtraction equation.
3. Starting with the deposit of $23 again, if you withdraw all $23, how much do you have left in your balance?
4. Write this scenario out as a subtraction equation.
5. Starting with the deposit of $23 again, if you withdraw $25, how can you indicate what has happened?
6. Write this scenario out as a subtraction equation. Use a negative sign in front of the number if it is representing debt.

Section B

For the rest of the activity, we will use green bills to represent deposits, and red bills to represent withdrawals.

1. Start with a balance of $0. Then, deposit $5 and deposit $7.
   (a) Count out green bills for each deposit. What is the new balance?
   (b) Write this scenario out as an addition equation.
2. Start with a balance of $0. Then, withdraw $3 and withdraw $6.
   (a) Count out red bills for each withdrawal. What is the new balance?
   (b) Write this scenario out as an addition equation. Remember to use a negative sign in front of the number if it is representing a withdrawal or debt.

3. Start with a balance of \$0. Then, deposit \$1 and withdraw \$1.
   (a) Count out a green bill for the deposit and red bill for the withdrawal. What is the new balance?
   (b) Write this scenario out as an addition equation.
   (c) Write this scenario out as an subtraction equation.
4. Start with a balance of \$0. Then, deposit \$4 and withdraw \$4.
   (a) Count out green bills for the deposit and red bills for the withdrawal. What is the new balance?
   (b) Write this scenario out as an addition equation.
   (c) Write this scenario out as an subtraction equation.
5. Start with a balance of \$0. Then, deposit \$8 and withdraw \$3.
   (a) Count out green bills for the deposit and red bills for the withdrawal. What is the new balance?
   (b) Write this scenario out as an addition equation.
   (c) Write this scenario out as an subtraction equation.
6. Start with a balance of \$0. Then, deposit \$6 and withdraw \$10.
   (a) Count out green bills for the deposit and red bills for the withdrawal. What is the new balance?
   (b) Write this scenario out as an addition equation.
   (c) Write this scenario out as an subtraction equation.
7. Start with a balance of \$0. Then, deposit \$34 and withdraw \$29.
   (a) Write this scenario out as an addition equation.
   (b) Write this scenario out as an subtraction equation.
8. Start with a balance of \$0. Then, withdraw \$48 and deposit \$89. Write this scenario out as an addition equation.
9. Start with a balance of \$0. Then, withdraw \$79 and deposit \$65. Write this scenario out as an addition equation.

10. Start with a balance of \$0. Then, withdraw \$15 and withdraw \$71.
    (a) Write this scenario out as an addition equation.
    (b) Write this scenario out as an subtraction equation.
11. Start with a balance of \$0. Then, withdraw \$84 and deposit \$68. Write this scenario out as an addition equation.
12. Start with a balance of \$0. Then, withdraw \$21 and deposit \$37. Write this scenario out as an addition equation.
13. Start with a balance of \$0. Then, withdraw \$12, then withdraw \$45, and finally withdraw \$17.
    (a) Write this scenario out as an addition equation.
    (b) Write this scenario out as an subtraction equation.
14. Start with a balance of \$0. Then, deposit \$99 and withdraw \$110.
    (a) Write this scenario out as an addition equation.
    (b) Write this scenario out as an subtraction equation.
15. Explain how to add negative numbers together with negative numbers.
16. Explain how to add positive numbers together with negative numbers.
17. Explain how to subtract a larger number from a smaller number.
18. Describe the mistake in the following solution of the difference $87 - 125$. Then, find the real difference:

$$\begin{array}{rr} & 87 \\ - & 125 \\ \hline & -62 \end{array}$$

5.9 Signed Numbers on a Number Line

When times are good, there is no need for subtraction:

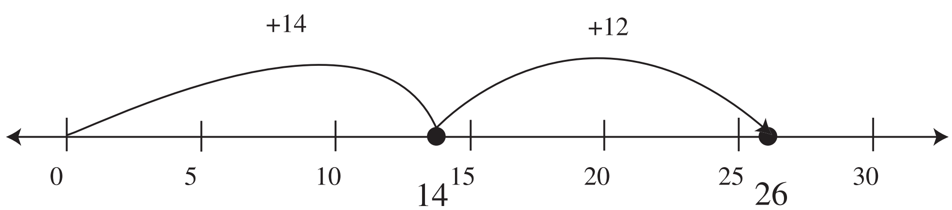
Example 1: Mel earned \$14 one day, and then earned another \$12 the next day. How much did he earn in all?

Solution: Mel's total earnings is the sum:

$$14 + 12 = 26$$

So, Mel earned a total of \$26.

We can illustrate this on a number line:



But as we all know, it's quite likely Mel will have to spend too. This is where subtraction comes in!

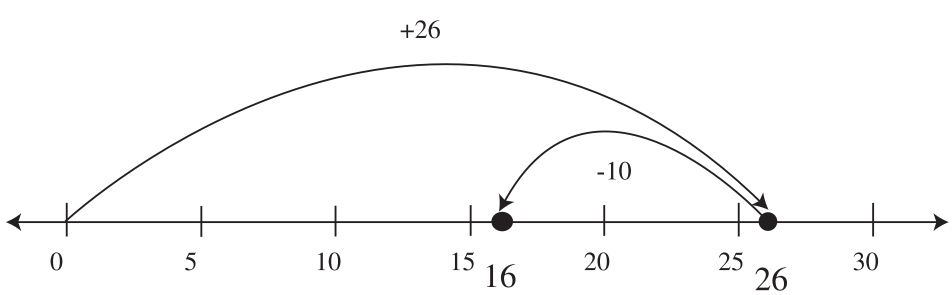
Example 2: Mel earned a total of \$26, then spent \$10. How much did he have left?

Solution: Mel had the difference between what he started with and what he spent, that is:

$$26 - 10 = 16$$

So, Mel had \$16 left.

We can illustrate this as well on a number line:



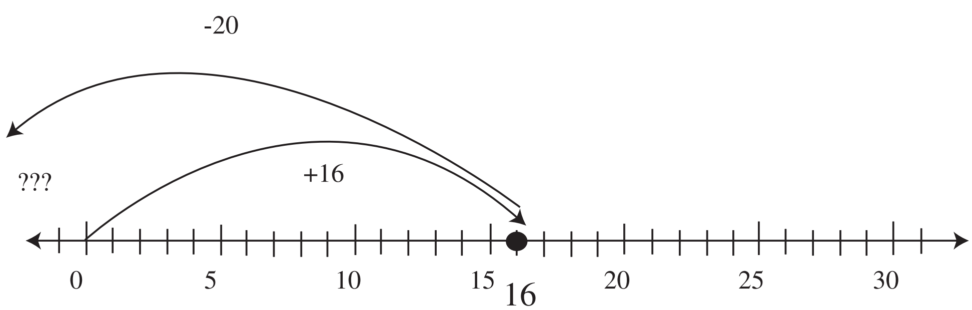
Finally, there is the situation that many of us run into. We spend more than we have!

Example 3: Mel had \$16 and then spent \$20. How much did he have left?

Solution: Again, Mel had the difference between what he started with and what he spent, that is:

$$16 - 20 = ???$$

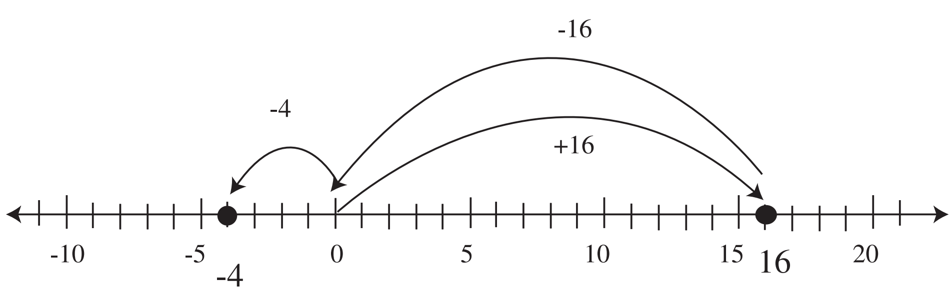
We can try to illustrate this as well on a number line, but run out of money. We subtract past 0!:



One way to express when you are in debt, that is, when you have spent more than you have, is to use **negative numbers**. If we think of starting with \$16 and then spending the \$20 is steps, first spend \$16 (until we are down to \$0), then continuing to spend the other \$4 putting us in debt \$4, we can think of the problem as:

$$\begin{aligned} 16 - 20 &= 16 - 16 - 4 \\ &= 0 - 4 \\ &= -4 \end{aligned}$$

Or, to see it on a number line:

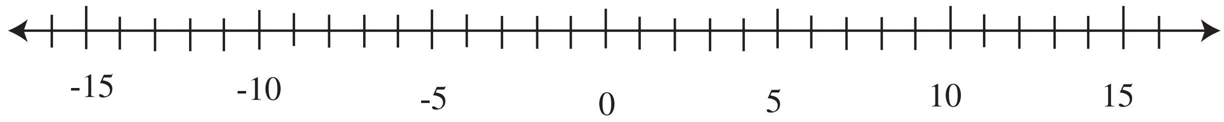


We use the negative symbol (which looks like a subtraction symbol) to represent that we are \$4 in **debt**.

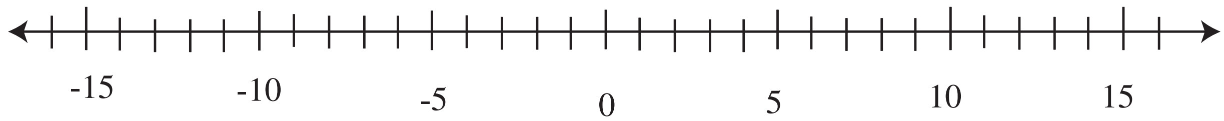
Exercise 5.9.1 Subtracting on a Number Line:

Illustrate the following subtractions on the given number lines.

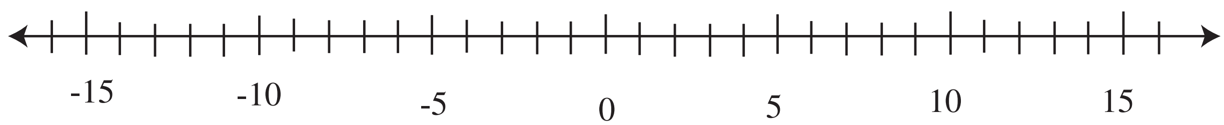
1. (a) $8 - 3$:



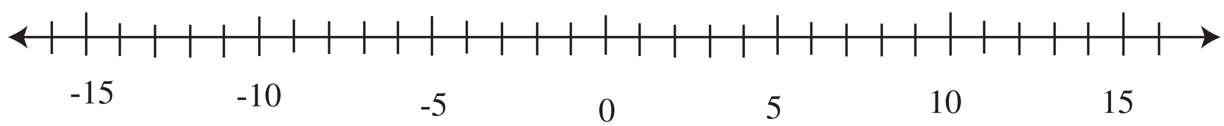
   (b) $3 - 8$:



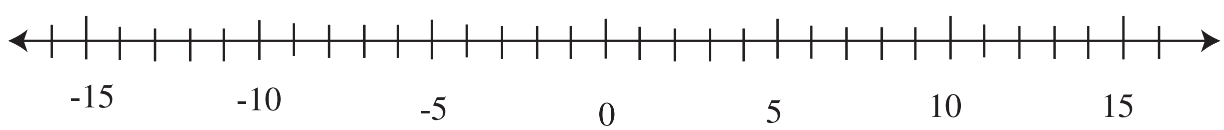
2. (a) $12 - 5$:



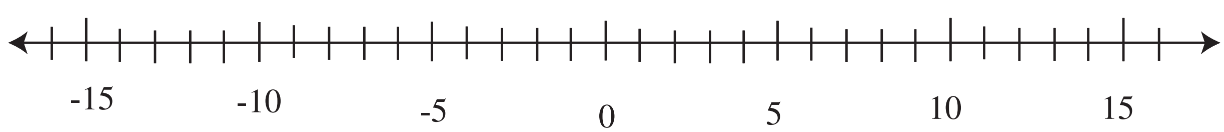
   (b) $5 - 12$:



3. (a) $17 - 11$:



   (b) $11 - 17$:



Exercise 5.9.2 Subtracting on a Calculator:

Find the following differences using your calculator.

1. (a) $28.32 - 15.95 =$ (b) $15.95 - 28.32 =$
2. (a) $47,320 - 39,115 =$ (b) $39,115 - 47,320 =$
3. (a) $2,485.50 - 1,926.69 =$ (b) $1,926.69 - 2,485.50 =$
4. (a) $1312.55 - 741.09 =$ (b) $741.09 - 1312.55 =$
5. (a) $0.236 - 0.091 =$ (b) $0.091 - 0.236 =$
6. (a) $123,456.789 - 98,765.4321 =$ (b) $98,765.4321 - 123,456.789 =$

Exercise 5.9.3 Investigating Subtracting:

1. Fill in the following table. You are given a starting amount of money and how much you are spending. You must fill in a subtraction problem that models the situation, as well as the ending amount. Use a negative symbol to indicate when you are in debt, that is, when you have spent more than you start with. The first two lines are done for you.

Starting Amount (in dollars)	Amount Spent (in dollars)	Subtraction Problem	Ending Amount (in dollars)
7	4	$7 - 4$	3
6	8	$6 - 8$	-2
10	6		
8	2		
5	9		
3	8		
14.2	12.8		
12.8	14.2		

2. Explain how to find the answer to a subtraction problem when the amount you are taking away is larger than the amount you start with.

Exercise 5.9.4 Compute the following. Include appropriate units in your answers:

1. $28 - 17$
2. $7 - 11$
3. $132 - 256$
4. $13.8 - 15.1$
5. $17.5 - 128$
6. $\frac{3}{2} - \frac{5}{2}$
7. $\frac{4}{3} - \frac{2}{3}$
8. $\frac{7}{8} - 1\frac{3}{8}$
9. After earning \$148.17, Paula spent \$152.08. What is her net worth?
10. Collin was driving to Death Valley in California from Bakersfield. He looked at an elevation sign in Bakersfield and it read, "Elevation: 505 feet". By driving to Death Valley, his elevation dropped by a total of 745 feet. What is the elevation in Death Valley?
11. On the same trip, Collin kept track of the temperature as well. While he was in Bakersfield, the Patella Credit Union digital clock flashed a temperature of $55°$ Fahrenheit. When he arrived in Death Valley, he looked at the thermometer on the gas station wall where he filled up his tank, and he realized that the temperature had dropped by $68°$ Fahrenheit! What was the temperature at that Death Valley gas station?
12. At Skyline College in San Bruno, the temperature was $58°$ Fahrenheit at 10 am when Julie arrived for her classes. When she left at 2 pm, the fog had lifted and it was $72°$ Fahrenheit. How much had the temperature increased while Julie was on campus?
13. Does it matter what order numbers are subtracted? For example, is $8 - 5$ the same as $5 - 8$? Explain. Describe what is the same and what is different in two subtraction problems where the only thing that has changed is the order that the numbers are written.

5.10 Order of Operations and Grouping Symbols

When the only operation is addition, the order and grouping of the numbers are added doesn't change the sum. We can use this fact to help make a sum of many numbers easier.

Example 1: Suppose the members of the Math Club decided to sell popcorn in the stands at the college basketball game as a fund-raiser. At the end of the game, the members brought their money collected together to be totaled. Michael collected \$27, Min collected \$29, Marleen collected \$23, Maurice collected 36, and Maria collected \$31. What was the total amount collected?

Solution: We could get the total by adding this in the order given:

$$\begin{aligned} 27 + 29 + 23 + 36 + 31 &= 56 + 23 + 36 + 31 \\ &= 79 + 36 + 31 \\ &= 115 + 31 \\ &= 146 \end{aligned}$$

so that the total is \$146.

We can make this easier, however, if we change the order that the numbers are written, and then group them in easy pairs:

$$\begin{aligned} 27 + 29 + 23 + 36 + 31 &= 27 + 23 + 29 + 31 + 36 \\ &= (27 + 23) + (29 + 31) + 36 \\ &= 50 + 60 + 36 \\ &= 110 + 36 \\ &= 146 \end{aligned}$$

which is the same total. We can add any way we want to!

We have seen, however, that with subtraction the order matters. For example,

$$6 - 4 = 2$$

while

$$4 - 6 = -2$$

Changing the grouping, that is, changing which operation comes first, also matters. For example,

$$7 - 3 + 1 = 4 + 1 = 5$$

while

$$7 - (3 + 1) = 7 - 4 = 3$$

Example 2: We just calculated that the Math Club collected $146 selling popcorn. Suppose, however that the popcorn maker costs $36 to rent for the night, and the supplies (popcorn, oil, salt, and bags to hold the popped popcorn) cost $19. Which of the following expressions can be used to calculate how much profit the Math Club made?

Solution 1: $146 - 36 + 19 = 110 + 19 = 129$, so the profit is $129,

or

Solution 2: $146 - (36 + 19) = 146 - 55 = 91$, so the profit is $91?

The first solution says we start with the $146, take away $36 and then add $19, implying that the $19 is extra money coming in rather than an expense. In the second solution, we total all of the expenses *before* taking it away from the income. Which is correct?

Exercise 5.10.1 Find the following.

1. $10 - 3 + 5$
2. $16 - 23 + 2$
3. $21 + 27 - 18 + 5$
4. $10 - (3 + 5)$
5. $16 - (23 + 2)$
6. $21 + (27 - 18) + 5$
7. $21 + 27 - (18 + 5)$
8. Write two different expressions for the following, then compute the value of each of the expressions to show that they both answer the question: Gina was keeping track of her balance in her checkbook. She started with a balance of $48.95, then made a deposit of $120. She then wrote checks for $25, $17.22, and $36.50. What was her final balance?