Fundamentals of Math
An Activities and Applications Approach

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HOW TO BE SUCCESSFUL IN A MATH CLASS

(regardless of your past experiences)

Success isn't a spectator sport. You succeed because you commit to your decision to do something and you act on that decision. People don't succeed because they're lucky, they succeed because they do something about it. You may have done well in math classes in the past or you may have had some difficulty but in the end the message is the same. If you want to learn math, if you want to pass this class, then you have to make it a conscious decision and follow a plan to see it though.

Although there are many reasons you may not have met your goals, there are also multiple strategies for succeeding in your next math class, especially if you begin with some good habits you maintain throughout the semester. From the beginning of the class, almost every day, through to the end of the semester, a successful student will follow most or all of the following:

- **Come to class.** This seems obvious, but it's critical. If you could do the work without attending class, you wouldn't be taking the class in the first place.
- **Do all the assigned work.** If you ever do get behind, get help immediately so that you can catch up with the current topics of each class.
- **If any of the material is unclear, get help from your instructor, a tutor, or a classmate.** The longer you wait to do this, the more the amount of unclear material builds up until virtually nothing makes sense in class anymore.
- **Study outside of class with classmates.** You may pick up ideas from them, and its easier to motivate for a study session if other people are counting on you.
- **Visit the Learning Center** to study in a structured environment with a tutor.
- **Be honest with yourself.** Only you know if you really understand a concept. Only you know if you have truly been doing your best. It's easy to make an excuse to yourself as to why you can't do something. Successful people persevere despite the obstacles in their way.

One last thing before we begin. Many students who have started the semester hating math have reported starting to enjoy it after working hard enough to start understanding the material. They confess that once they began doing the math correctly with confidence, the work actually became fun! So, I say to you, "Get busy, get help, and have fun!"

HOW TO USE THIS BOOK

The book in your hands is meant to be used. At the end of the semester it will be full of your work, your notes, your sweat, and your tears. Whether your Fundamentals of Math class allows the students to use their notes on exams or not, you need the work in the book to be correct, clear to read, and organized so that you can easily refer to previous work to figure out how to do a problem

Some of the exercises or activities in the book don't leave enough space to show a clear solution.

Some of the exercises or activities in the book require that you work with physical objects, reflect on what you have learned, then summarize the ideas in clear sentences.

Some of the exercises or activities in the book ask you to fill in a table.

Often you will be asked to explain how something works rather than being told in the text how it works.

All of these require that you figure out how to show your work in such a way that both you and your instructor can clearly understand what you have done. The following suggestions may help you to find a way to do this for yourself.

- If there is very little space to show your work in the book, write your solution on a separate piece of paper.
- Any work you write on a separate piece of paper, label carefully with the page number and exercise number so that it is easy to see where it goes. Write on paper that is 3-hole punched so that you can put it in your binder next to the page it refers to.
- Do your work and write your notes on scratch paper first, then when you have checked that it is correct and checked it for clarity, re-write it as neatly as you can before putting it together in the book.

Activity 0.1 Problem Solving

Objective: To find a solution to a problem by working in a group. To share problem solving strategies with the class.

Materials: None.

Group size: 3 to 4.

Instructions: First solve the problem clearly by writing out all of your steps, then answer the questions below.

You are invited to your boss's house at 8:00 pm. At 6:30 pm you have just left the candy shop in your neighborhood where you picked up a box of candy for a hostess gift. You notice that your favorite store is having a sale. You have had your eye on a designer leather jacket for a while now. There is a line outside the store and the manager says there cannot be more than 10 people in the store at any time. You decide to wait in the line. There are 22 people ahead of you. People are leaving the store at a rate of 2 people every 5 minutes and once you get in the store, it takes you 10 minutes to find the jacket in the right size and 15 minutes to pay for it. Will you make it to your boss's house on time? Don't forget you have a 12 minute drive from the shopping center to her house.

1. What is the answer to the problem?

2. Show clearly all of the steps used to find the answer.

3. What did you do to check whether the answer is correct? Does everyone in the group agree that it is correct?

4. What pictures did you draw to help you solve the problem?

5. How did you organize the information on paper so that you could communicate your ideas clearly to your group-mates?

6. How can you write your solution clearly enough so that you will be able to follow your steps long after you've forgotten what the problem was about?

Notes:

1 Numbers

1.1 Defining Numbers

Numbers have two main uses in our world: to identify something the same way that a name does, and to represent a value. Some examples of identification numbers are account numbers, phone numbers, address numbers, social security and G-numbers, and titles or names like Chapter 5 or R2D2. Numbers that represent value are much more common, and are the subject of the study of mathematics. Some examples are anything to do with money like prices or wages, scores in sports or on an exam, measurements like area, distance, weight or speed, a time or date, and age. Often, value numbers are used to compare one object with another. For example, a jacket which costs $45 with another that costs $100; a job that offers a salary of $25,000 per year with a job that offers a salary of $40,000 per year; an apartment with an area of 500 square feet with one with an area of 800 square feet. People use value numbers to make decisions, and mathematics helps people make informed decisions.

Activity 1.1 Introducing Units: How can the meaning of the same number be different?

Objective: To learn how the meaning of numbers change based on the units assigned to them.

Materials: None.

Group size: 3 to 4.

Instructions: Use the numbers 10, 16, 0, 21, and 5 to answer the first four questions, then work on the last two.

1. Discuss together what each number in the list above reminds you of. Notice that numbers just by themselves are a very abstract concept. When numbers quantify **SOMETHING** or describe a quantity of **SOMETHING**, called a unit, they are more meaningful. Therefore Units are associated with numbers. For example: 10 *yards*, is quite different from 10 trees.
2. By yourselves, assign different units to each of the numbers 10, 16, 0, 21, and 5.
3. When everyone is done, compare your answers.
   (a) Which of the numbers did more than one person have the same units for?
   (b) Explain why that might have happened. Did those numbers HAVE to have those units to make sense, or was it just a common thing for that number to stand for?
4. Explain why it is important to include proper units with answers to word problems.
5. Tonight at home, search a newspaper for three examples of identification numbers and three examples of value numbers. For the value numbers, write a sentence explaining what object in real life is being represented with that value and include the units if there are any.
6. Tomorrow at the beginning of class, find your group members and discuss what you found.

1.2 Comparing Numbers

What determines if one number is greater than another?

We use the symbol "$>$" to indicate when one number represents more of something than another number. For instance, since 8 pounds of something is more than 5 pounds, we would write 8 lbs. $>$ 5 lbs. We use "$<$" to indicate when one number represents less of something than another number. From the same example above, we would write 5 lbs. $<$ 8 lbs.

Exercise 1.2.1 To search for the answer to this question, let's do the following exercise:

1. For each question insert either $\boxed{>}$, $\boxed{<}$, or a $\boxed{?}$ if it is unclear whether one quantity is more or less than the other.
   (a) 10 dogs $\square$ 7 dogs
   (b) 10 dogs $\square$ 7 cats
   (c) 10 dogs $\square$ 7 buckets
   (d) \$5 $\square$ 34 ¢
   (e) 2 feet $\square$ 20 inches
   (f) 12 miles $\square$ 3 gallons
   (g) \$42,000 per year $\square$ \$5,000 per month
   (h) \$12.95 per hour $\square$ \$2,500 per month
2. Which part(s) of question (1) could you clearly answer? Why?
3. What information do you need to answer the questions that were not clear? Why?
4. Explain your answers to parts (e) through (h).
5. Write a conclusion about what you need in order to compare two things using $>$ or $<$.

Activity 1.2 Powers of 10

Objective: To investigate the pattern when multiplying by powers of ten.

Materials: Calculator.

Group size: 2 to 5.

1. Use your calculator to help you find the products below.

Number of 10's	Expression	Result
2	$10 \times 10 =$	
3	$10 \times 10 \times 10 =$	
4	$10 \times 10 \times 10 \times 10 =$	
5	$10 \times 10 \times 10 \times 10 \times 10 =$	
6	$10 \times 10 \times 10 \times 10 \times 10 \times 10 =$	

2. Without using your calculator, anticipate the result of the product:

   $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 =$

3. Describe a shortcut for multiplying tens.

4. How many 10's are multiplied together to obtain the number $1,000,000,000$?

5. Find the results of the products below, then write the names of the results out in words. (Hint: For the decimals, think of money. For example, $0.1 = 0.10$ is a dime which is a tenth of a dollar, and 0.01 is a penny which is a hundredth of a dollar!)

	Expression	Result	Names in Words
(a)	$4 \times 100,000 =$	400,000	four hundred thousand
(b)	$6 \times 0.01 =$	0.06	six hundredths
(c)	$7 \times 1000 =$		
(d)	$2 \times 100 =$		
(e)	$3 \times 1000 =$		
(f)	$5 \times 10,000 =$		
(g)	$9 \times 10 =$		
(h)	$2 \times 0.1 =$		
(i)	$5 \times 0.1 =$		
(j)	$3 \times 0.01 =$		

6. What is the name of the number 407,200?

   Notice how the name of a number combines its digits with the power of ten implied by the position of the digit. The 4 represents $4 \times 100,000 = 400,000$, the 7 represents $7 \times 1000 = 7,000$ and the 2 represents $2 \times 100 = 200$.

7. What is the name of the number 53,090?
8. How much does the 5 represent in the number 53,090?
9. How much is 10 quarters worth?
10. How much is 10 dimes worth?
11. How much is 100 pennies worth?
12. How much is 1000 pennies worth?
13. How much is 1000 dimes worth?

2 Place Value

2.1 Digits

There are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Numbers are represented by placing digits together in a string. The digits in a number have different values associated with them depending on their relative position in the number.

For example:

> 276 has three digits and 7 has a value of **seven***ty* or $7 \times 10 = 70$. As you read the number, the place values are spoken as, **two** *hundred* **seven***ty*-**six**.
>
> A decimal point separates whole number values from fractional values.
> 24.89 has 4 digits and the digit 8 has the value: $8 \times 0.1 = 0.8$. For example, if it was \$24.89, the 8 would represent 8 dimes, or $8\times$\$0.1=\$0.80.

Exercise 2.1.1 Associating digits with money. \$1 bill, \$10 bill, \$100 bill, \$1000 bill

Leticia wants to buy a new motorcycle that costs \$6,327.95

1. How many digits are in 6,327.95?
2. Which is the largest digit?
3. Which digit has the largest place value?
4. What is the value of the digit with the largest value?
5. Which is the smallest digit?
6. Which digit has the smallest value and what is its value?
7. Leticia pays cash for the bike and the bills she can get from the bank are in the denominations \$10,000, \$1000, \$100, \$10, \$1, as well as dimes and pennies. How many of each denomination will she need to cover the exact price if she uses the minimum number of bills?
8. The value of each denomination of the bills Leticia used in question (7) is the place value of each digit within the number 6,327.95. Name the place value of each digit.

Note: As you see in exercise 2.1.1, there is a monetary value associated with each digit of a number according to the placement of the number. For example, the value of the digit 6 is 6 thousand since you need 6 one thousand dollar bills and the value of the 5 is 5 hundredths because you need 5 pennies and each penny is worth one hundredth of a dollar.

2.2 Introducing exponents with base 10

The following chart shows some of the place values in the number system:

one billion	$1,000,000,000 = 1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^9$
one hundred million	$100,000,000 = 1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^8$
ten million	$10,000,000 = 1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^7$
one million	$1,000,000 = 1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^6$
one hundred thousand	$100,000 = 1 \times 10 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^5$
ten thousand	$10,000 = 1 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^4$
one thousand	$1,000 = 1 \times 10 \times 10 \times 10 = 1 \times 10^3$
one hundred	$100 = 1 \times 10 \times 10 = 1 \times 10^2$
ten	$10 = 1 \times 10 = 1 \times 10^1$
one	$1 = 1 \times 1 = 1 \times 10^0$
one tenth	$0.1 = \frac{1}{10} = 1 \div 10$
one hundredth	$0.01 = \frac{1}{100} = 1 \div 100 = 1 \div 10 \div 10$
one thousandth	$0.001 = \frac{1}{1000} = 1 \div 1000 = 1 \div 10 \div 10 \div 10$

The following chart illustrates the place value of the digits:

one billion		one hundred million	ten million	one million		one hundred thousand	ten thousand	one thousand		one hundred	ten	one		one-tenth	one-hundredth	one-thousandth
	,				,				,				.			

Therefore, the number three billion, four hundred seventy five million, six hundred twenty eight thousand, five hundred eleven and nine-tenths (3,475,628,511.9) would look like this in the chart:

one billion		one hundred million	ten million	one million		one hundred thousand	ten thousand	one thousand		one hundred	ten	one		one-tenth	one-hundredth	one-thousandth
3	,	4	7	5	,	6	2	8	,	5	1	1	.	9		

Exercise 2.2.1

1. Find the place value and **value** of each digit underlined in following chart:

	Number	Place Value	Value
(a)	$23\underline{7},669$	thousands	7,000
(b)	$\underline{9},932,210$		
(c)	$1\underline{0}2$		
(d)	$86\underline{5},106$		
(e)	$65.2\underline{9}3$		
(f)	$1\underline{0}.226$		
(g)	$0.\underline{2}26$		
(h)	$\underline{6}50,900,563,115.07$		
(i)	$1127.0\underline{8}335$		

2. What number is written as forty two thousand, three hundred seventy five and twenty three hundredths?

3. What number is written as eighty billion nine hundred seven thousand four?

4. A “googol” is defined as 10^{100}, that is, 10 multiplied by itself 100 times. Describe how you could write this number on paper in standard notation.

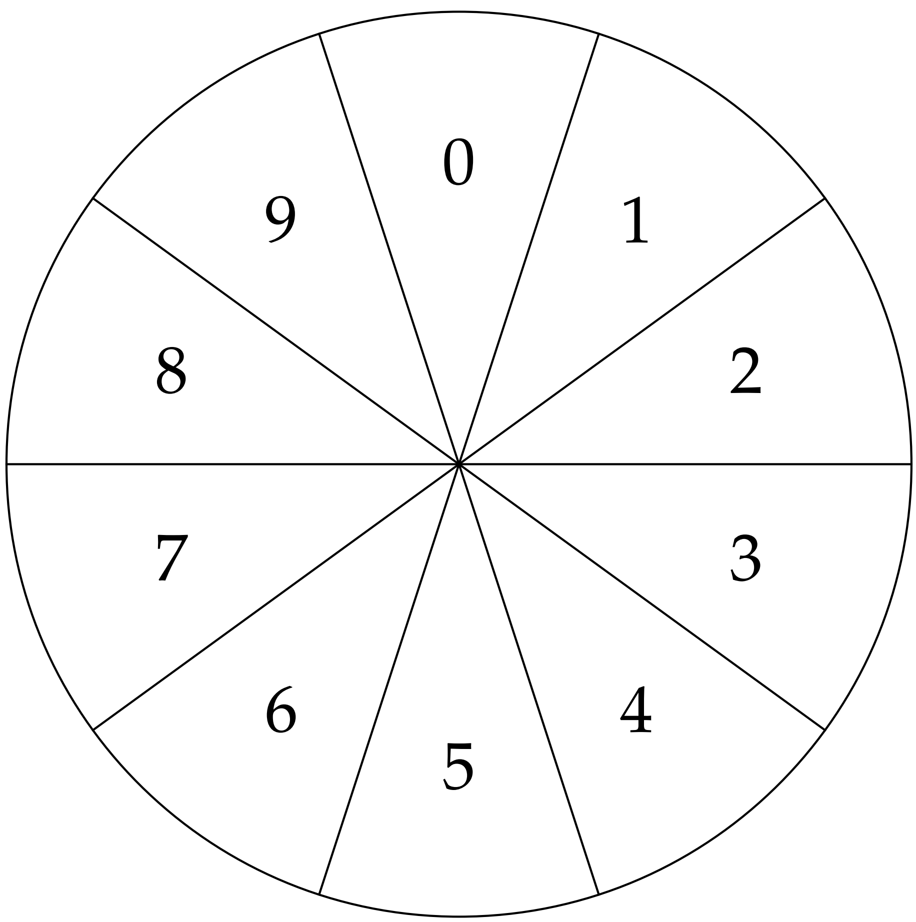
Activity 2.1 Group Game: Spin the Digits

Objective: To introduce concepts of place value by strategizing digit placement.

Materials: Paper clip, paper, pencil.

Group size: 3 to 4.

1. Each group needs a paper clip and a separate piece of paper for each person.
2. Open the paper clip from one side and use it as a spinner.
3. Each member draws 3 lines on his or her paper.
4. Each time one of the members spins the paper clip and each member records the number on one of the 3 lines that he or she has drawn.
5. After 3 spins compare your 3-digit numbers.
6. The largest 3-digit number wins the turn.



7. What is a good strategy for winning this game?

Activity 2.2 Learning Place Value with Money

Objective: To solidify concepts of place value by using money.

Materials: Bills of different denominations.

Group size: 3 to 4.

Notice that $235 could come in 235 $1 bills, or 2 $100 bills, 3 $10 bills and 5 $1 bills , or 23 $10 bills and 5 $1 bills.

1. Which combination has the fewest number of bills totaling $235? (Write how many of each denomination in the appropriate boxes.)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

2. Find three different combinations of bills/coins to make $400. (Write how many of each denomination in the appropriate boxes.)

(a)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

(b)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

(c)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

3. Find three different combinations of bills/coins to make $1. (Write how many of each denomination in the appropriate boxes.)

	$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
(a)				,				.		

	$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
(b)				,				.		

	$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
(c)				,				.		

4. Find three different combinations of bills/coins to make $7,000. (Write how many of each denomination in the appropriate boxes.)

	$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
(a)				,				.		

	$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
(b)				,				.		

	$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
(c)				,				.		

5. Find three different combinations of bills/coins to make \$5,628. (Write how many of each denomination in the appropriate boxes.)

(a)

\$100,000	\$10,000	\$1,000		\$100	\$10	\$1		\$0.10	\$0.01
			,				.		

(b)

\$100,000	\$10,000	\$1,000		\$100	\$10	\$1		\$0.10	\$0.01
			,				.		

(c)

\$100,000	\$10,000	\$1,000		\$100	\$10	\$1		\$0.10	\$0.01
			,				.		

6. Find three different combinations of bills/coins to make \$63,921.42. (Write how many of each denomination in the appropriate boxes.)

(a)

\$100,000	\$10,000	\$1,000		\$100	\$10	\$1		\$0.10	\$0.01
			,				.		

(b)

\$100,000	\$10,000	\$1,000		\$100	\$10	\$1		\$0.10	\$0.01
			,				.		

(c)

\$100,000	\$10,000	\$1,000		\$100	\$10	\$1		\$0.10	\$0.01
			,				.		

7. Find three different combinations of bills/coins to make $307,995. (Write how many of each denomination in the appropriate boxes.)

(a)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

(b)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

(c)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

8. Raymond had 21 - $1000 bills, no $100 bills, no $10 bills, 26 - $1 bills, no dimes and 13 pennies. He went to the bank and exchanged them for the *fewest* number of bills/coins that represented the same amount. How much of each kind of bill/coin did he get? In the bank record below, record the value of Raymond's money and the quantity of each type of bill/coin he would have when using the fewest number of each to represent the amount.

Type of Bill/Coin	$10,000s	$1,000s	$100s	$10s	$1s	$0.1s (dimes)	$0.01s (pennies)
Count of Bill/Coin	________	________	________	________	________	________	________

How much would his money be worth?

9. What part (fraction) of $1 is a dime?

10. What part (fraction) of $1 is a penny?

11. What part (fraction) of $1 is a quarter?

2.3 Making Numbers

Exercise 2.3.1

1. When writing money in dollars and cents format, where do you put the decimal place? (between the count of which of the bills/coins?)
2. Using each digit exactly once, what would be the largest number you could make with the digits 0, 1, 2, 5, 8, 9 (with no decimal points)?
3. Why did you put the left-most digit in that spot?
4. Using each digit exactly once, what would be the smallest number you could make with the digits 0, 1, 2, 5, 8, 9 (with no decimal points)?
5. Why can't you put the smallest digit in the left-most place?
6. Using each digit exactly once, what would be the smallest 6-digit number with 2 digits after the decimal point you could make with the digits 0, 1, 2, 5, 8, 9?
7. Using each digit exactly once, what would be the largest 6-digit number with 2 digits after the decimal point you could make with the digits 0, 1, 2, 5, 8, 9?
8. If you had the digits 0, 1, 2, 5, 8, 9 to make a 6-digit number, using each digit exactly once, what would be the smallest number you could make with the 6 digits? You choose the number of digits after the decimal place.
9. What is the largest 5-digit number?
10. What is the smallest 5-digit number (without the decimal point)?
11. Using the digits 6, 4, 1, 0, and 8, exactly once each, what is the largest five digit number you can make?
12. Using the digits 6, 4, 1, 0, and 8, exactly once each, what is the largest five digit number you can make? You may include a decimal point.

2.4 Place Value and Value:

Exercise 2.4.1

1. In the number \$3,461.85, which digit has the highest place value (i.e., accounts for the most money?)
2. In the number \$2.08, which digit has the highest place value?
3. In the number \$2.08, which is the largest digit? What is its value?
4. In the number \$2.08, which digit has the highest value? What is its value?
5. In the number \$4501.32, what is the place value of the digit 0?
6. In the number \$673.47, what is the place value of the digit 4?
7. In the number \$86, 793.64, what digit is in the ten-thousands place?
8. In the number \$86, 793.64, what digit is in the hundreds place?
9. In the number \$86, 793.64, what digit is in the tenths place?
10. In the number \$86, 793.64, what is the place value of the digit 9?
11. In the number \$86, 793.64, what is the value of the digit 8?
12. In the number \$86, 793.64, what is the place value of the digit 4?
13. In the number \$86, 793.64, what is the value of the digit 4?
14. Create a number with a 9 in the thousands place.
15. Create a number with a 3 in the tens place and a 5 in the tenths place.
16. Create a number with a 2 in the hundreds place, a 1 in the hundredths place, and the digit in the ones place is twice the digit in the tens place.

2.5 Saying and Writing Numbers in Words

To read and write whole numbers: The whole numbers are the numbers in the sequence 0, 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, continuing indefinitely. When we write whole numbers in standard form we separate a group of three digits by a comma. Each group of three digits forms what is called a **period**. Each period has a particular name. We say the name of a period after we read all the numbers in that period, except the last period before the decimal point.

Example 1: Read 8,412,769,215.

Solution: Eight *billion*, four hundred twelve *million*, seven hundred sixty nine *thousand*, two hundred fifteen. (The name of each period is in *italics*. Notice that the last period doesn't have a name.)

To read and write decimal numbers: We read the whole number part exactly the same way as before, then read the decimal as "and" followed by the name of the number to the right of the decimal point using the place value of the last digit as the number's value.

Example 2: Read 65,389,237,542.029.

Solution: Sixty five billion, three hundred eighty nine million, two hundred thirty seven thousand, five hundred forty two, and twenty nine thousandths. (Notice that the name of the decimal part is *thousandths* since the last digit, the 9, is in the thousandths place.)

Exercise 2.5.1 Place commas to separate the periods, then write out the number in words:

1. 2863 ______________________________
2. 76.218 ______________________________
3. 28990.02 ______________________________
4. 3400651899 ______________________________
5. 23678.93 ______________________________
6. 1000267 ______________________________
7. 64822.2 ______________________________

Exercise 2.5.2

1. What number is 10 more than 5,687.13?
2. What number is 400 more than 38,265.79?
3. What number is 0.3 more than 21,975.52?
4. What number is 0.05 more than 5,687.13?

Notes:

3 Rounding, Estimation, and Measurement

3.1 Rounding

Some quantities are exact, like the number of people in a room, while others are *approximations*. An approximation is a number that is close to the exact value, but has fewer non-zero digits. More zeros makes it easier to work with.

We **round** a number to a less accurate but simpler approximate value to make it easier to work with, while keeping most of the important information about the number.

For example, if we learned that the U.S. trade deficit in June 2005 was $58,753,421,989, we probably could analyze how this was important to our lives just as well if it was rounded to $59,000,000,000! In fact, without all of the non-zero digits, we could even write this number as a ”word-number” hybrid by writing, ”$59 billion”. It is usually a good idea to round a number like this, and if you listen to financial news reports, you will notice that the media often rounds numbers.

For another example, if we wanted to measure the length of a window so that we could buy the right size replacement, we could use a very accurate measuring device and find that the length is 75.18952 cm. The window store is probably satisfied with the rounded version, 75 cm. But, the window is approximately 75 cm.

Exercise 3.1.1 For the following numbers, state whether it seems reasonable that the given value is exact or an approximation. Give a reason for your answer.

1. Measurement of the distance you drive from SF to LA is 420 miles.
2. Jenisa calculated her share of the lunch that she had with her high school friends yesterday and the figure was $12.345.
3. The family’s rent each month was $1225.
4. The same family’s monthly budget for food was $840.
5. A group of friends calculated their share of the cost of a ski trip and the figure was $342.

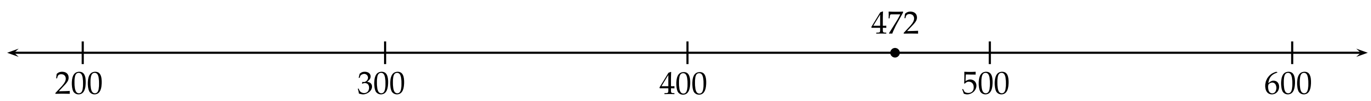
Exercise 3.1.2 For the following numbers, discuss how accurate each measurement needs to be for it to be useful, and why it requires that amount of accuracy. Also use phrases like “at least” or ”at most” or “to the nearest” if appropriate for the situation.

1. Measurement for rope to tie down a load.
2. Measurement for wood in making a picture frame.
3. A cup of flour to make a cake.
4. Measurement of the length of the glass used to make the windows for a new building.
5. Scores used to compute your grade.
6. The amount of line to let out in a cast while fishing.
7. The time it will take to drive to a dinner party.
8. The amount of salt used in a pot of soup.

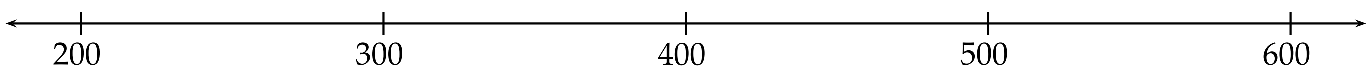
3.2 Graphing Numbers on a Number Line

Exercise 3.2.1 Graph the following numbers on the number line.

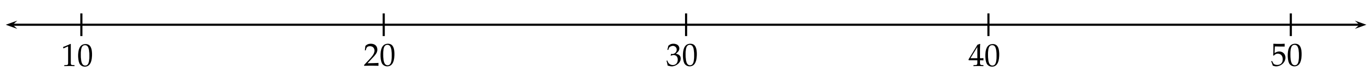
Example: To graph 472, place a dot where 472 should go, then write 472 next to the dot as in the figure:



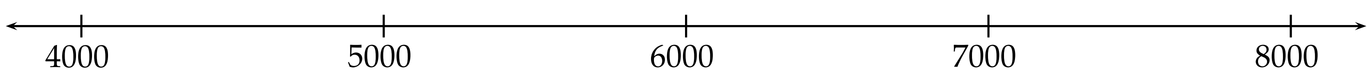
1. Graph 220, 517, 350, 490:



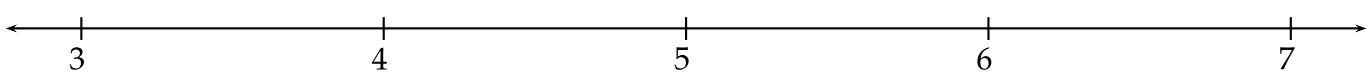
2. Graph 28, 17, 35, 46:



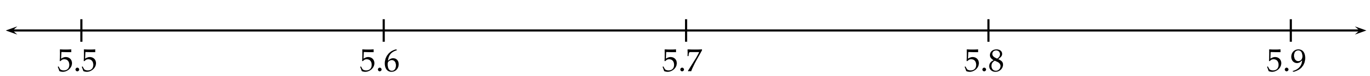
3. Graph 5593, 7200, 6500, 4207:



4. Graph 3.5, 5.2, 6.6, 4.1:

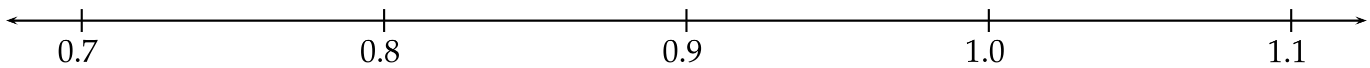


5. Graph 5.72, 5.63, 5.88, 5.55:

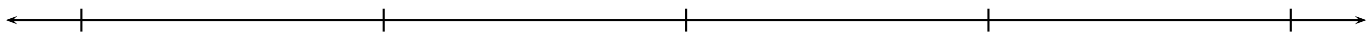


Exercise 3.2.2 Mark each number line by counting by the indicated amount.

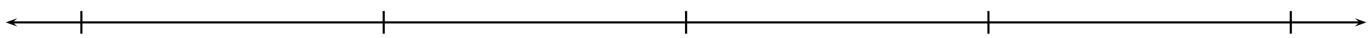
Example: To mark the number line by counting by tenths starting at 0.7, you would mark the number line like the following:



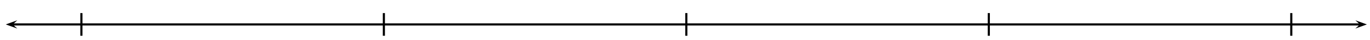
1. Count by hundreds, starting at 500:



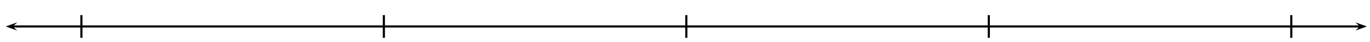
2. Count by tens, starting at 30:



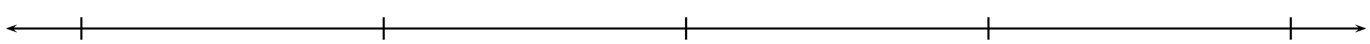
3. Count by ones, starting at 8:



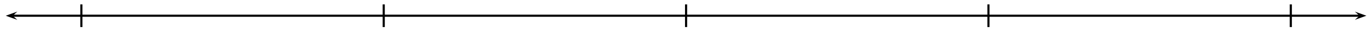
4. Count by tens, starting at 420:



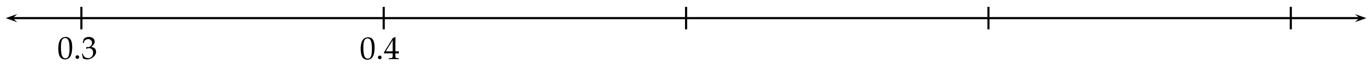
5. Count by hundreds, starting at 2100:



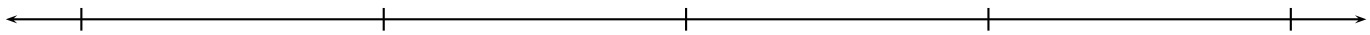
6. Count by tens, starting at 2100:



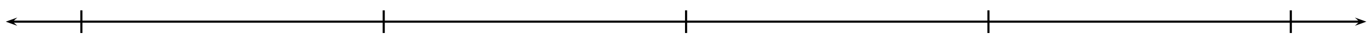
7. Count by tenths (0.1s), starting at 0.3:



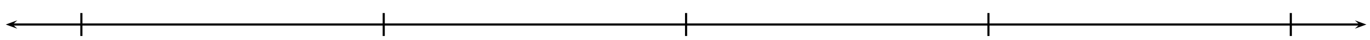
8. Count by tenths, starting at 1.8:



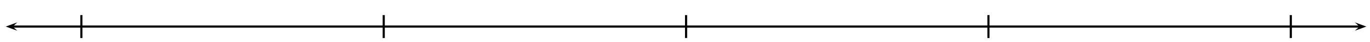
9. Count by tenths, starting at 22.5:



10. Count by hundreds, starting at 34,800:



11. Count by thousands, starting at 998,000:



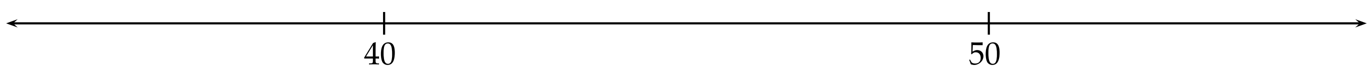
3.3 Rounding to a Particular Decimal Place:

To round numbers to a particular decimal place, do the following:

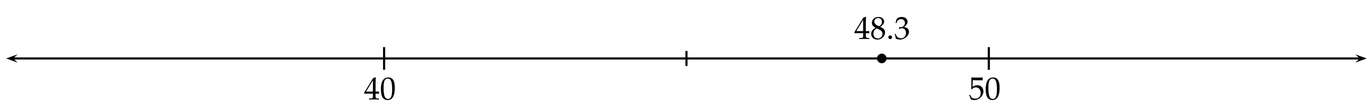
1. Determine the correct decimal place (tenths, ones, tens, hundreds, etc.)
2. Draw a number line, and count by the determined decimal place starting at the number just below the number you are rounding, and ending with the number just after it.
3. Graph the number that you are rounding on the number line.
4. The rounded value is the count number that is closest to your graphed number. If the graphed number is exactly in the middle of two count numbers, round to the higher number.

Example: Round 48.3 to the nearest ten.

Solution: To round 48.3 to the nearest ten, count by tens starting at 30 like the following:



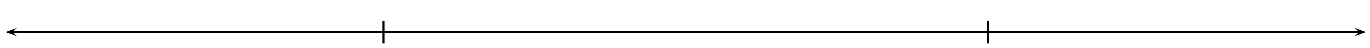
Then, graph 48.3, and determine which number is closest...it helps to put a tick mark at the half-way points:



48.3 is to the right of the half-way point between 40 and 50, so it's closer to 50. Therefore, 48.3 rounded to the nearest ten is 50.

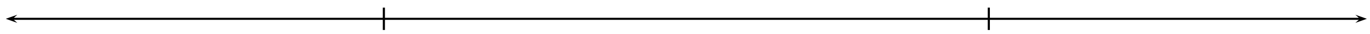
Exercise 3.3.1 Round the following as indicated.

1. Round 286 to the nearest hundred:



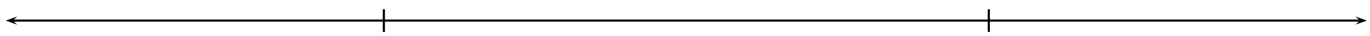
Final answer:_______________

2. Round 5.37 to the nearest one:



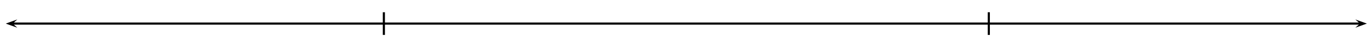
Final answer:________________

3. Round 5.37 to the nearest tenth:



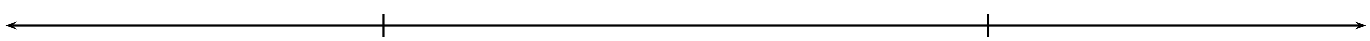
Final answer:________________

4. Round 5.37 to the nearest ten:



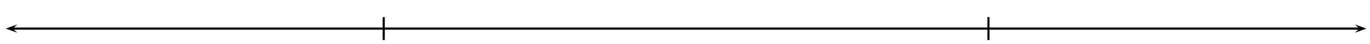
Final answer:________________

5. Round 5.37 to the nearest hundred:



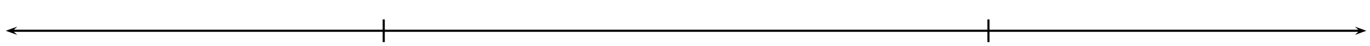
Final answer:________________

6. Round 2358 to the nearest hundred:



Final answer:________________

7. Round 2358 to the nearest thousand:



Final answer:________________

Exercise 3.3.2 Fill in the table to practice rounding. The first two are done for you as examples:

The Number to Round off and the Level of Rounding	**Underline the Digit(s)**	**The Next Digit**	**Round Up or Down**	**Answer**
1) (example) 18.09 Round to the nearest 10	1̲8.09	8	Up, since $8 \geq 5$	20
2) (example) 23.032 Round to the nearest 100th	23.03̲2	2	Down, since $2 < 5$	23.03
3) 5671.98 Round to the nearest 100				
4) 5671.983 Round to the nearest 100th				
5) 1098.67 Round to the nearest 100				
6) 36.0409 Round to the nearest 100th				
7) 99.9986 Round to the nearest 1				
8) 99.9986 Round to the nearest 10th				
9) 0.00098 Round to the nearest 100th				
10) 6.85 Round to the nearest 10th				

Exercise 3.3.3 For each table of data, make a new table by rounding values as indicated:

1. The following table lists the number of pages in each of the books in the Harry Potter series. Fill in the new table with the number of pages rounded to the nearest 10.

Given Data Table

Book Number	**Number of Pages**
1	309
2	341
3	435
4	734
5	870
6	652

Rounded Data Table

Book Number	**Number of Pages**
1	
2	
3	
4	
5	
6	

2. The following table lists the numbers of new products containing the artificial sweetener Splenda for various years. Fill in the new table with the number of new products containing Splenda rounded to the nearest 100.

Given Data Table

Year	Number of New Products Containing Splenda
2000	183
2001	261
2002	365
2003	561
2004	1330

Rounded Data Table

Year	Number of New Products Containing Splenda
2000	
2001	
2002	
2003	
2004	

3. The following table lists the percentages of Americans who have earned a college degree for various years. Fill in the new table with the percentages of Americans who have earned a college degree rounded to the nearest whole percentage.

Given Data Table

Year	Percentage of Americans Earning a College Degree
1960	7.7
1970	10.7
1980	16.2
1990	21.3
2000	25.6
2003	27.2

Rounded Data Table

Year	Percentage of Americans Earning a College Degree
1960	
1970	
1980	
1990	
2000	
2003	

4. The more you stretch a spring with your hands, the more *force* the spring exerts on your hands. The following table compares the amount of stretch with the forces exerted. Fill in the new table with the amount of stretch data rounded to the nearest 100th.

Given Data Table

Force (newtons)	Stretch (meters)
0.5	0.018
1.0	0.035
1.5	0.052
2.0	0.069
2.5	0.087
3.0	0.104
3.5	0.121
4.0	0.139
4.5	0.156
5.0	0.173

Rounded Data Table

Force (newtons)	Stretch (meters)
0.5	
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	

3.4 Estimation and Measurement

Exact or Estimate

When you are finding out the value of a quantity in real life (as opposed to reading a given value in a book), some values you know to be **exact**, while others you must **estimate** to obtain an **approximation**. Some examples of exact values are as follows:

- The number of people at a small dinner party.
- The number of children in a family.
- The number of hours in a day.
- The price of gas at a gas station.
- The number of grams in a kilogram.
- The number of units passed in a semester.

By contrast, the following is a list of values that must be estimated:

- The time it takes to complete an assignment.
- The number of people who live in San Francisco.
- The weight of a sick dog at the vet.
- The distance from SF to LA.
- A person's height.

Exercise 3.4.1

1. Explain what makes the values in the first list exact values, and the values in the second list approximate values.
2. State five things that have a value that is exact.
3. State five things that have a value that must be estimated.
4. For the following quantities, give a value for the quantity, then state whether it is exact or an estimation:

   (a) The number of people in class on the most recent day.

   (b) The height of the classroom door.

   (c) The length of your index finger.

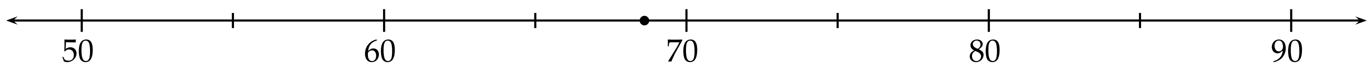
   (d) The number of typed words on this page.

   (e) The weight (in pounds) of the dog you have most recently seen.

Estimating Points on a Number Line

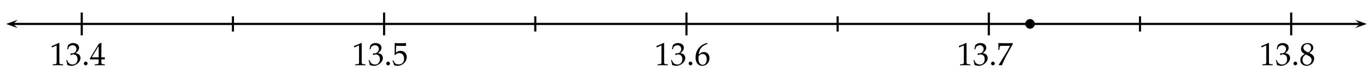
Exercise 3.4.2 For the following, use the number lines to estimate the value of the marked point.

Example 1:



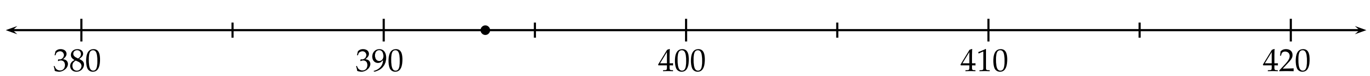
Solution: The value of the point is approximately 68. Since the value of the marked numbers are counting by tens, we only have confidence in one more decimal place. Therefore, we estimate to the nearest whole number.

Example 2:

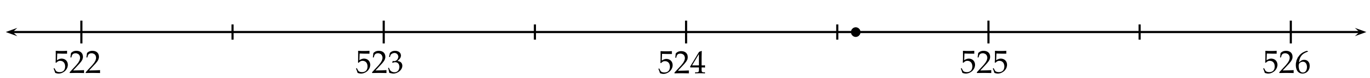


Solution: The value of the point is approximately 13.72. Since the value of the marked numbers are counting by tenths, we only have confidence in one more decimal place. Therefore, we estimate to the nearest hundredth.

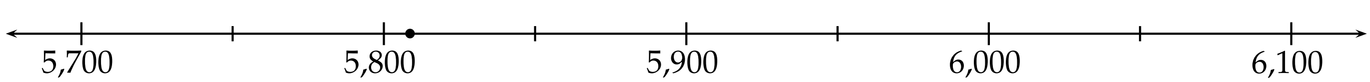
1.



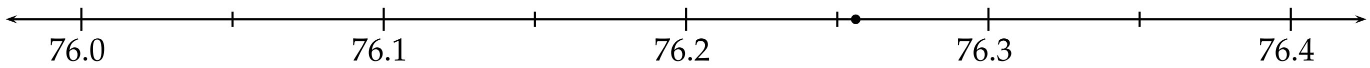
2.



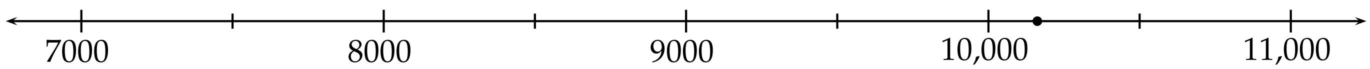
3.



4.



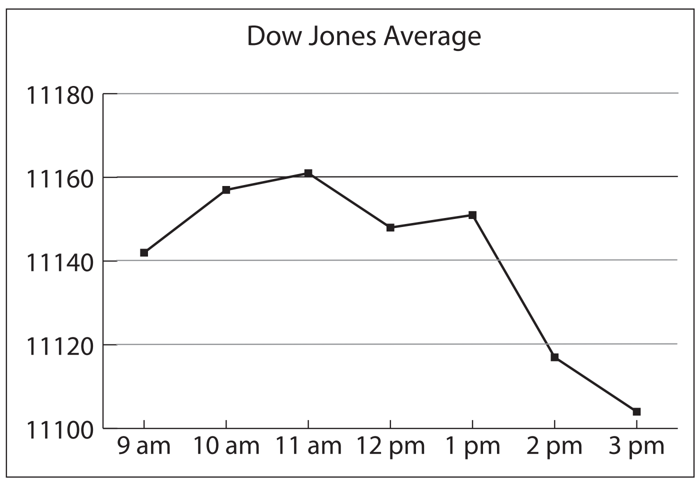
5.



Estimating Values from Line and Bar Graphs

Exercise 3.4.3 Estimate the values from the graphs.

1. Use the following graph to estimate the value of the Dow Jones Average at certain times.



Example: Estimate the value of the Dow at 10 am.

Solution: The value of the Dow at 10 am is estimated to be 11,158.

(a) Estimate the value of the Dow at 9 am.

(b) Estimate the value of the Dow at 11 am.

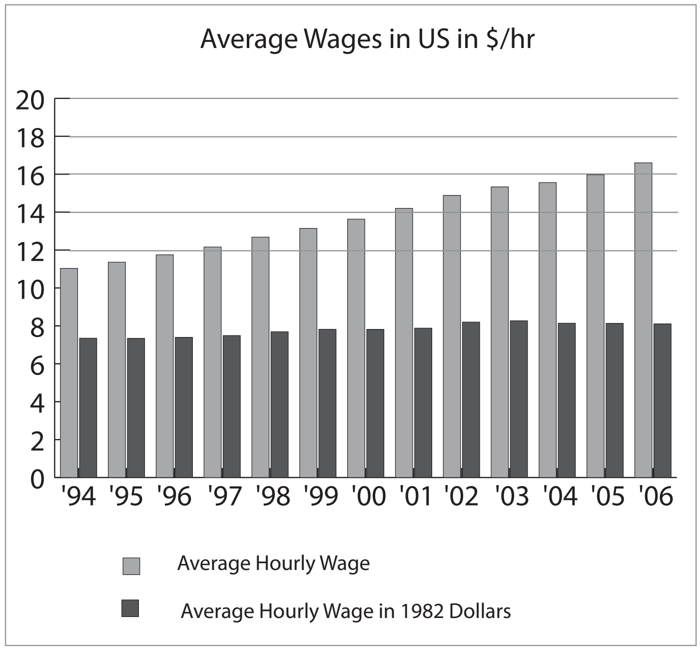
(c) Estimate the value of the Dow at 12 pm.

(d) Estimate the value of the Dow at 1 pm.

(e) Estimate the value of the Dow at 2 pm.

(f) Estimate the value of the Dow at 3 pm.

2. Use the following graph to estimate the average U.S. wage (in dollars per hour) in June of different years.



Example: Estimate the average U.S. wage in June 1997.

Solution: The average U.S. wage in June 1997 is estimated to be $12.10 per hour.

(a) Estimate the average U.S. wage in June 1994.

(b) Estimate the average U.S. wage in June 1996.

(c) Estimate the average U.S. wage in June 1999.

(d) Estimate the average U.S. wage in June 2001.

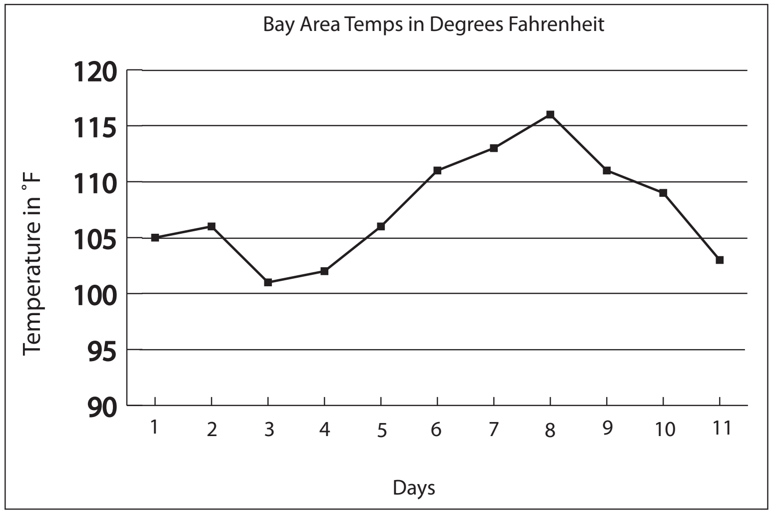
(e) Estimate the average U.S. wage in June 2004.

(f) Estimate the average U.S. wage in June 2006.

(g) When were wages about $15 per hour?

(h) What do the "Ave. Wage in 1982 Dollars" bars tell you?

3. Use the following graph to estimate the high temperatures during a record breaking eleven day heat wave in the Bay Area.



(a) Estimate the high temperature on day 1.

(b) Estimate the high temperature on day 3.

(c) Estimate the high temperature on day 6.

(d) Estimate the high temperature on day 7.

(e) Estimate the high temperature on day 10.

(f) On what day was the highest Bay Area high temperature recorded?

(g) On what day was the lowest Bay Area high temperature recorded?

(h) Between which days is the temperature going up?

An Introduction to Measurement

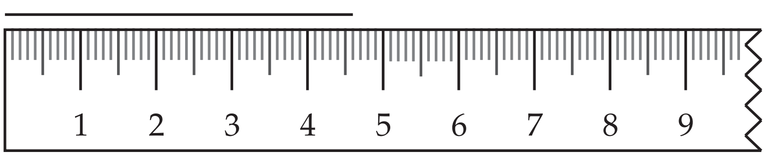
When we are estimating the length of something, we can get a more accurate measurement by using a tool. One might use a ruler to measure the length of a finger, a tape measure to measure the length of a window sill, and an odometer to measure the distance from SF to LA. In the U.S., the units of length are the inch, the foot, the yard, and the mile. We measure with units that are appropriate for the lengths. For example, we measure the distance between two cities in miles, the length of a book in inches, and the length of a room in feet. In this section, you will be using the **metric** system to measure length, and the most appropriate metric system unit to use for the following lengths is the **centimeter**.

We could use an inch ruler, but inches are broken down into fractions of an inch rather than tenths. We will wait to introduce inches until after we have introduced fractions.

To use a ruler to measure length, line the end of the ruler at one end of the line, and note how far along the ruler the other end of the line is.

Example 1: Measure the length of the following line accurate to one-tenth of a centimeter:

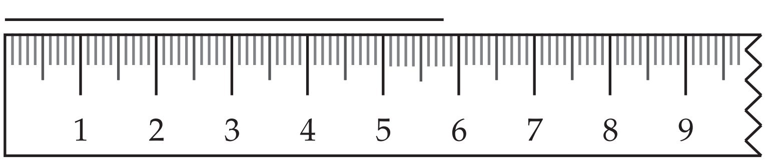
Solution: We place the ruler near to the line as follows:



Looking at where the line ends, we estimate the length as 4.6 cm. The 4 stands for 4 whole centimeters which is indicated by the 4 on the ruler. The .6 (6 tenths) stands for 6 out of ten parts of the next centimeter. We know this because the end of the line is above the 6th mark past the 4.

Example 2: This time, measure the length of the following line accurate to one whole centimeter:

Solution: We place the ruler near to the line as follows:



Since the end of the line is closer to 6 cm than it is to 5 cm, we say the line measures approximately 6 cm accurate to the nearest whole centimeter.

Exercise 3.4.4 Using a centimeter ruler, measure the lengths of the following lines accurate to the nearest whole centimeter. Include appropriate units in your answers.

1.

2.

3.

4.

5.

Exercise 3.4.5 Using a centimeter ruler, measure the lengths of the following lines accurate to the nearest one-tenth of a centimeter. Include appropriate units in your answers.

1.

2.

3.

4.

5.

4 Addition of Whole Numbers, Decimals, and Fractions

4.1 Totaling Money:

Activity 4.1 Learning Concepts of Addition with Money

Objective: To solidify concepts of addition by using money.

Materials: Bills of different denominations.

Group size: 3 to 4. If there are 4, one of them acts as banker.

Write the names of each group member:

Group member #1 ________________________
Group member #2 ________________________
Group member #3 ________________________
Group member #4 ________________________

In this activity each member of your group will begin with a specified amount of money. Assume that the only bills (and coins) available are in denominations of $10,000, $1000, $100, $10, $1, dimes and pennies. Each of the three members in your group take bills/coins from the pack to total the following amounts:

- Member #1: $5,693.12
- Member #2: $9,275.35
- Member #3: $2,968.78

When you take your money, make sure that the number of bills/coins in each denomination is fewer than ten. For each of the following problems, start each member with their original amount of money and with their original number of each type of bill/coin.

1. (a) If you put the first two members' stacks of money together in one pile, how many of each bill /coin do member #1 and member #2 have together? Record the number of bills/coins in each denomination.

$\$10,000$	$\$1000$	$\$100$	$\$10$	$\$1$	$0.10	$0.01

   (b) Exchange bills with the bank, wherever possible, so that the total amount takes the fewest bills necessary. e.g., twelve $100 bills should be exchanged for two $100 bills and one $1000 bill. Record the new number of bills/coins in each denomination.

$\$10,000$	$\$1000$	$\$100$	$\$10$	$\$1$	$0.10	$0.01

   (c) How much money do they have together?

2. (a) If you put the first and third members' stacks of money together in one pile, how many of each bill/coin do member #1 and member #3 have together? Record the number of bills/coins in each denomination.

$10,000	$1000	$100	$10	$1	$0.10	$0.01

(b) Exchange bills with the bank, wherever possible, so that the total amount takes the fewest bills/coins necessary. Record the new number of bills/coins in each denomination.

$10,000	$1000	$100	$10	$1	$0.10	$0.01

(c) How much money do they have together?

3. (a) How many of each bill/coin do member #2 and member #3 have together? Record the number of bills/coins in each denomination.

$10,000	$1000	$100	$10	$1	$0.10	$0.01

(b) Exchange bills with the bank, wherever possible, so that the total amount takes the fewest bills/coins necessary. Record the new number of bills/coins in each denomination.

$10,000	$1000	$100	$10	$1	$0.10	$0.01

(c) How much money do they have together?

4. (a) How many of each bill/coin do all three members have together? Record the number of bills/coins in each denomination.

\$10,000	\$1000	\$100	\$10	\$1	\$0.10	\$0.01

(b) Exchange bills/coins with the bank, wherever possible, so that the total amount takes the fewest bills/coins necessary. Record the new number of bills/coins in each denomination.

\$10,000	\$1000	\$100	\$10	\$1	\$0.10	\$0.01

(c) How much money do they have together?

5. Explain how you found the combined monies (sums) above.

6. Compare your answers with another group in your class. Compare the number of bills/coins in each denomination from each question before and after combining monies. Are there any common ways of totaling that your groups followed?

7. (a) Add 27 and 15 using the traditional paper and pencil method.

(b) Add 27 and 15 using the paper money method, that is by counting total bills and exchanging any that go over ten bills.

(c) Compare "carrying the one" from the traditional method and "exchanging denominations that are greater than ten bills" from the paper money method.

Activity 4.2 Addition Activities

Objective: To practice adding in your head. No calculators please!

Materials: Paper, pencil dice.

Group size: 3 to 4.

Activity 1: Pig

Each group chooses a number between 30 and 55. Each player may roll the die as many times as she likes. The player then adds the face value of die to the sum so far and tries to get as close to the chosen target number as possible. Here's the catch: if a player rolls the same number two times in a row, they lose their turn and their points! (For a variation: the player keeps their points so far and just loses their turn). For example, a roll might look like:
$3 + 5 + 6 + 2 + 5 + 4 + 4$. The player loses their points and their turn because they rolled two fours in a row before stopping or reaching the target. It is possible to win this game with a roll of 1 — if the other players go out first.

Activity 2: Value of Words

By assigning a value to each of the letters of the alphabet, all of a sudden, words have value aside from their ability to help us communicate. For the problems below, use the values given in the chart for each letter. The value of a word is the total of the values of its letters.

A=$1	G=$7	L=$12	Q=$17	V=$22
B=$2	H=$8	M=$13	R=$18	W=$23
C=$3	I=$9	N=$14	S=$19	X=$24
D=$4	J=$10	O=$15	T=$20	Y=$25
E=$5	K=$11	P=$16	U=$21	Z=$26
F=$6				

1. Show that "Skyline" is worth $95.
2. How much is "Pacifica" worth?
3. Who has the most expensive name (last or first; not both)?
4. Find a 3-letter word that has:
   (a) the cheapest value
   (b) the most expensive value
5. What is the most expensive word that you can find?
6. What do the following words have in common? Explain.
   acknowledge, beginnings, carpenter, delivery, elsewhere, friendlier, governs, hospital, immature, judiciary

Exercise 4.1.1 Complete the following addition table, then continue to answer the questions:

+	0	1	2	3	4	5	6	7	8	9	10
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											

1. Find at least four different patterns in this table. An example of a pattern could be that starting from the upper left and going down diagonally to the right, the numbers start at zero and go by 2's.

2. Notice that there are two ways to get 2 as a sum of two numbers: 0 + 2 (we'll assume 2 + 0 is the same) and 1 + 1.

   (a) How many different ways are there to get the number 3 as a sum of two numbers? ________

   (b) How many different ways are there to get the number 4 as a sum of two numbers? ________

   (c) How many different ways are there to get the number 5 as a sum of two numbers? ________

   (d) How many different ways are there to get the number 6 as a sum of two numbers? ________

   (e) How many different ways are there to get the number 7 as a sum of two numbers? ________

   (f) How many different ways are there to get the number 8 as a sum of two numbers? ________

   (g) How many different ways are there to get the number 9 as a sum of two numbers? ________

3. What patterns do you notice in your answers to question 2? (List at least three). Is there an easy way to use the table to find the different sums? Explain.

4. How many different ways are there to get the number 10 as a sum? ____________

5. From your observations in the previous questions you should be able to guess the number of different ways to write 100 as a sum:

6. Complete the following addition table:

+	4	9	6	5	1	10	3	7	0	2	8
1											
3											
5											
8											
4											
9											
7											
10											
2											
0											
6											

7. Complete the following addition table:

+	3					8			0	2	
5							15				9
					3			7			
						16					
3				12							
		8						6		3	
								14			
6			12								
						18					
	10										
											8
								5			

Vocabulary:

- The operation is called **addition**.
- Each of the numbers that we add together is called an **addend**.
- When two numbers are added together, the answer is called the **sum** or **total**.

4.2 The Addition Process

During the money activity at the beginning of the chapter, you observed that when we add money, we combine the bills with the same denominations together and every time the number of bills in each denomination reaches 10, we replace that with one bill from the next higher denomination, that is, the one to the left of it in the place value table. For example, ten dimes is replaced by 1 dollar or ten $10 bills is replaced by one $100 bill.

Group Discussion Activity

Suppose you worked as a mechanic at a gas station, and were totaling the charges for a customer. Suppose the charge for the parts came to $247 and the labor charge was $78. If your calculator was not handy, you might use pencil and paper to write the following:

$$\begin{array}{r} {}^{1}{}^{1} \\ 247 \\ 78 \\ \hline 325 \end{array}$$

1. Why are the numbers lined up the way they are instead of like this:

$$\begin{array}{l} 247 \\ 78 \\ \hline \end{array}$$

2. Where do the small ones on top of the 2 and the 4 in 247 come from? What do they signify?
3. Is there a way to approximate the answer without using pencil and paper and without using a calculator? Explain your process.

Adding whole numbers or decimals together:

1. Line up each addend so that the digits with the same place values are in the same column.
2. Start adding from the rightmost column.
3. If the sum of the numbers in a column is 9 or less, record the sum and proceed to the next column.
4. If the sum of the digits is 10 or more, write the rightmost digit of the sum under the column you are adding and add the other digits to the appropriate columns to the left of it. This amount is "carried over" to the next higher place value column.
5. Repeat until all the digits of all the columns are added.

Example 1: Add the following two numbers:

$25,693.12 + 9,275.3$

Solution:

First, line up the numbers so that **each place value is in the same column**, that is, the pennies are all in the same column, the dimes in the next, etc. Since there are no pennies in $9,275.3, we place a zero in the hundredths (pennies) column for that number.

ten-thousands, thousands, hundreds, tens, ones, dimes, pennies

$$\begin{array}{r} 25{,}693.12 \\ 9{,}275.30 \\ \hline \end{array}$$

Now, we start totaling by column, starting with the pennies. There is a total of 7 pennies, 4 dimes, and 8 dollar bills. Each of these totals is 9 or less, so no need to exchange bills. At this point the problem looks like:

$$\begin{array}{r} 25{,}693.12 \\ 9{,}275.30 \\ \hline 8.42 \end{array}$$

When we get to the tens place, the total of 9 and 7 is 16. We exchange 10 of the tens to get one extra hundred. We place a small 1 on the top of the hundreds column to remind us of the extra hundred, and put the remaining 6 tens in the tens column of the answer.

$$\begin{array}{r} {\scriptstyle 1} \\ 25{,}693.12 \\ 9{,}275.30 \\ \hline 68.42 \end{array}$$

We next proceed to the hundreds place, and with the extra hundred we have a total of 9. With 14 thousands, we have to carry 10 of them for an extra ten-thousand. The final solution should look similar to this:

$$\begin{array}{r} {\scriptstyle 1}{\scriptstyle 1} \\ 25{,}693.12 \\ 9{,}275.30 \\ \hline 34{,}968.42 \end{array}$$

Therefore, the answer is 34,968.42.

Example 2: Add the following three numbers:

$76,926.8 + 685.87 + 93,768.79$

Solution:

Following the steps, we obtain a solution that looks similar to:

$$\begin{array}{r} {}^{1\,2\,1\,2\,2\,\,1} \\ 76{,}926.80 \\ 685.87 \\ \underline{93{,}768.79} \\ 171{,}381.46 \end{array}$$

Therefore, the answer is 171,381.46.

Exercise 4.2.1 Add the following numbers without using a calculator:

1. $\begin{array}{r} 23,409 \\ +\quad 12,000 \\ \hline \end{array}$

2. $\begin{array}{r} 678,222.2 \\ +\quad 7,392.3 \\ \hline \end{array}$

3. $\begin{array}{r} 731.03 \\ +\quad 2.006 \\ \hline \end{array}$

4. $\begin{array}{r} 987,962.051 \\ 16 \\ +\quad 687.084 \\ \hline \end{array}$

5. $11,276 + 0.038 + 2.38$

6. $99 + 10,268.5 + 28.03$

7. $0.00397 + 2.015 + 69.11$

8. $763.19 + 2,763.019 + 3.0019$

9. $287,309,227.56 + 199.99$

10. $0.48 + 268 + 60.46$

4.3 Estimation of Sums

The purpose of estimating a sum before calculating it exactly is to:

- Give you a value that is relatively close to the actual sum.
- Make the calculation much easier to do. Often, the estimation process is so easy, you can do it in your head!

Often each of these two points contradict each other. The easier the calculation, the farther the estimate is from the actual.

Activity 4.3 Investigating Estimation Techniques

Objective: To investigate options for estimating sums.

Materials: None.

Group size: 3 to 4.

Technique 1: Rounding all numbers to the largest place value of the larger number.

- Note the largest place value in the larger number.
- Round each number to the place value noted.
- Add the rounded numbers.

Example 1: Estimate the sum:

$5,629 + 236$

Solution:

5,629 is the larger number, and its largest place value is the thousand's place.

	5629	——>		6000
+	236	——>	+	0
				6000

The estimate of the sum of 5,629 and 236 using technique 1 is 6,000.

1. Using technique 1, estimate the sum $23,409 + 12,000$

   (a) What is the largest place value of the larger number?
   (b) Round 23,409 to the largest place value of the larger number.
   (c) Round 12,000 to the largest place value of the larger number.
   (d) Add together the rounded numbers from part (b) and (c).

2. Using technique 1, estimate the sum $678,222.2 + 7,392.3$

   (a) What is the largest place value of the larger number?
   (b) Round 678,222.2 to the largest place value of the larger number.
   (c) Round 7,392.3 to the largest place value of the larger number.
   (d) Add together the rounded numbers from part (b) and (c).

Technique 2: Rounding all numbers to the largest place value of the smaller number.

- Note the largest place value in the smaller number.
- Round each number to the place value noted.
- Add the rounded numbers.

Example 2: Estimate the sum:

$5,629 + 236$

Solution:

236 is the smaller number, and its largest place value is the hundred's place.

	5629	——>		5600
+	236	——>	+	200
				5800

The estimate of the sum of 5,629 and 236 using technique 2 is 5,800.

3. Using technique 2, estimate the sum $23,409 + 12,000$
   (a) What is the largest place value of the smaller number?
   (b) Round 23,409 to the largest place value of the smaller number.
   (c) Round 12,000 to the largest place value of the smaller number.
   (d) Add together the rounded numbers from part (b) and (c).

4. Using technique 2, estimate the sum $678,222.2 + 7,392.3$
   (a) What is the largest place value of the smaller number?
   (b) Round 678,222.2 to the largest place value of the smaller number.
   (c) Round 7,392.3 to the largest place value of the smaller number.
   (d) Add together the rounded numbers from part (b) and (c).

5. Estimate $731.03 + 2.006$ first by rounding all numbers to the largest place value in the larger number (technique 1), then by rounding both numbers to the largest place value in the smaller number (technique 2).
   (a) Estimate using technique 1:
   (b) Estimate using technique 2:

6. Estimate $987,962.051 + 16 + 687.084$ first by rounding all numbers to the largest place value in the larger number (technique 1), then by rounding both numbers to the largest place value in the smaller number (technique 2).
   (a) Estimate using technique 1:
   (b) Estimate using technique 2:

7. Estimate $11,276 + 0.038 + 2.38$ first by rounding all numbers to the largest place value in the larger number (technique 1), then by rounding both numbers to the largest place value in the smaller number (technique 2).
   (a) Estimate using technique 1:
   (b) Estimate using technique 2:

8. Estimate $99 + 10,268.5 + 28.03$ first by rounding all numbers to the largest place value in the larger number (technique 1), then by rounding both numbers to the largest place value in the smaller number (technique 2).
   (a) Estimate using technique 1:
   (b) Estimate using technique 2:

9. Estimate $0.00397 + 2.015 + 69.11$ first by rounding all numbers to the largest place value in the larger number (technique 1), then by rounding both numbers to the largest place value in the smaller number (technique 2).

   (a) Estimate using technique 1:
   (b) Estimate using technique 2:

10. Estimate $763.19 + 2,763.019 + 3.0019$ first by rounding all numbers to the largest place value in the larger number (technique 1), then by rounding both numbers to the largest place value in the smaller number (technique 2).

   (a) Estimate using technique 1:
   (b) Estimate using technique 2:

11. Estimate $287,309,227.56 + 199.99$ first by rounding all numbers to the largest place value in the larger number (technique 1), then by rounding both numbers to the largest place value in the smaller number (technique 2).

   (a) Estimate using technique 1:
   (b) Estimate using technique 2:

12. Estimate $0.48 + 268 + 60.46$ first by rounding all numbers to the largest place value in the larger number (technique 1), then by rounding both numbers to the largest place value in the smaller number (technique 2).

   (a) Estimate using technique 1:
   (b) Estimate using technique 2:

13. In general, on the sums where the two techniques give a different answer, which technique takes less work to compute the sum after the rounding step?

14. In general, on the sums where the two techniques give a different answer, which technique gives an estimate that is more accurate?

15. Describe the kind of sums that give the same estimate when using technique 1 as when using technique 2.

We have seen in the activity that technique 1 and technique 2 can give different answers. Also, we have seen that in general, technique 1 takes less work to do while technique 2 usually gives a more accurate answer. In general with estimation, we want to have a method that makes the sum easier to compute while still being relatively accurate. So, which technique is more appropriate?

Exercise 4.3.1 Choosing the Appropriate Technique

For the following situations,

(a) Estimate the sum using technique 1. Include appropriate units in your answers.

(b) Estimate the sum using technique 2. Include appropriate units in your answers.

(c) State which technique you think is more appropriate for the situation.

(d) Explain the reasons why you chose the technique that you did in part (c).

1. Lou is grocery shopping and he only has $80 left in his checking account. He wants to use his debit card to pay for the groceries, but doesn't want to be embarrassed and have to put some items back if he doesn't have enough money. The items in his cart and their prices are listed in the table below:

Item	**Price**	**Tech 1** $\approx$	**Tech 2** $\approx$
1 bag of frozen shrimp	$19.98		
1 package of beef ribs	$3.87		
1 box of Wheaties	$4.49		
1 12-pack of Negra Modelo	$14.99		
1 bottle of Pinot Noir	$21.99		
1 half-gallon of Horizon organic milk	$4.39		
1 pint of whipping cream	$4.49		
1 dozen eggs	$3.19		
1 bottle of Advil	$16.79		

Does he have enough money or does he have to put something back?

(a) Technique 1:

(b) Technique 2:

(c) Which technique is more appropriate?

(d) Why?

2. Marta is planning her family's monthly budget. The following table summarizes how much she plans to spend in different categories:

Category	**Budget**	**Tech 1 ≈**	**Tech 2 ≈**
Rent	\$854.00		
Car Payment	\$137.42		
Power, water, and garbage	\$93.75		
Phones	\$84.20		
Food	\$370.00		
Cable	\$75.00		
Netflix	\$17.99		
Credit Card Payment	\$175		

What is Marta's total monthly budget?

(a) Technique 1:

(b) Technique 2:

(c) Which technique is more appropriate?

(d) Why?

Exercise 4.3.2 Approximate Sum Practice

For each of the following exercises:
(a) Estimate the answers either by rounding all numbers to the largest place value in the largest number (technique 1), or by rounding both numbers to the largest place value in the smallest number (technique 2). Include appropriate units in your answers.

(b) State which technique you used to perform the estimate, and why you chose that technique over the other one.

(c) Find the actual sum. Include appropriate units in your answers.

(d) State whether the actual sum is $>$, $=$, or $<$ the estimate.

1. During a shopping spree, Margaret spent $259.05 at Costco and $72.99 at Ross. How much did she spend in all?

2. The Smiths recorded their mileage over the summer and were shocked to find they went 3,759.12 miles in June and 4,250.7 miles in July. They decided to stay home during the entire month of August, so only went 66 miles. What was their total distance traveled for the three months?

3. A mountain is 27,369 feet high. On the top of the mountain is a building that is 150 feet tall. On the roof of the building is a woman who is 6.13 feet tall. How high is the top of her head above sea level?

4. Mark lives at home to try to save money. He earns $24,269.5 per year. His dad earns $35,253.05 and his mom earns $41,299.8. What is Mark's family's total income each year?

5. After his cramp went away, Rick ran 5.099 miles on flat ground, 0.89 miles uphill, then 2.01 miles downhill. At the end of the downhill, his legs started cramping again! How far did Rick run between cramps?

6. A government spent $7,831,004.05 per day for the military, and $32,890.9 per day to fund the student loan program. How much did this government spend each day on the military and student loans?

7. Georgio bought a used car off Craig's list for $8,750 and then filled up the gas tank for $48.72. How much did he spend in all?

8. $2,224,766 + 5,689,201 + 1,920,761$

9. $2,256,703 + 156.56$

10. $276,107 + 24,109.5 + 3,655.7 + 390.7$

Activity 4.4 Magic Squares

Objective: To practice addition and puzzle solving.

Materials: Paper, pencil.

Group size: 3 to 4.

Look at the square grid of numbers shown below. See if you can find anything unusual about the numbers in the square.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

This square is called a magic square because the numbers in each row, in each column, and in each diagonal all add up to the same thing, called the magic number. In this case the magic number is 34. The artist Albrecht Dürer (1471 - 1528) is credited with being the first European to publish a magic square, by including the square shown above in his copper engraving, ”Melancholia I”. See if you can find the year the piece was completed, hidden somewhere in the magic square.

1. The magic square below is composed of the digits 1 through 9 (exactly once each) but some have been left out. Fill in the missing numbers in the correct places.

8		6
	5	
4		2

2. The numbers 1, 3, 7, 8, 9, 10, 14, and 15 have been removed from the magic square below. Put them back in the correct locations.

	14		4
11		6	
	10		5
13	2		

3. The numbers 6, 10, 11, 12, 13, 14, 15, and 16 have been removed from the magic square below. Put them back in the correct locations.

9		3	
4			5
	1	8	
7			2

4.4 Addition in Real Life

In a math class, we see the word "sum" used to indicate that we are supposed to add. In real life, the operation of addition can be implied in many other ways. Some key words to look for are total, increase, or perimeter, but sometimes none of these are used, and you have to reason, from the context, that addition is implied.

Example 1: Marco and Mary went Christmas shopping. They found an alarm clock for $24.99, a CD for $14.45, a pillow for $32.20, a rug for $18.69, and a game for $11.50. Estimate the pre-tax cost for all of these items by rounding each value to the nearest whole dollar before computing, then find the actual pre-tax cost.

Solution:

It's not stated explicitly in the problem, but we can reason that the cost for all of the items is the sum of each cost. We begin by rounding each to the nearest whole dollar rather than using another estimation technique since the problem explicitly says how it wants it done:

$$\$24.99 \approx \$25$$

$$\$14.45 \approx \$14$$

$$\$32.20 \approx \$32$$

$$\$18.69 \approx \$19$$

$$\$11.50 \approx \$12$$

Then, we add the rounded values, $25 + 14 + 32 + 19 + 12 = 102$, to get our estimate of $\$102$.

Finally, we add up the numbers with the cents included: $24.99 + 14.45 + 32.20 + 18.69 + 11.50 = 101.83$, which is close to our estimate, so we are confident that we didn't make a careless error! The pre-tax cost of all of the items is: $101.83.

Exercise A: Gracie and her husband were in New York City at a discount department store. They bought sheets for $34.96, shorts for $16.95, socks for $5.24, and a duvet for Gracie's mom for $48.32. Estimate the pre-tax cost for all of these items by rounding each value to the nearest whole dollar before computing, then find the actual pre-tax cost.

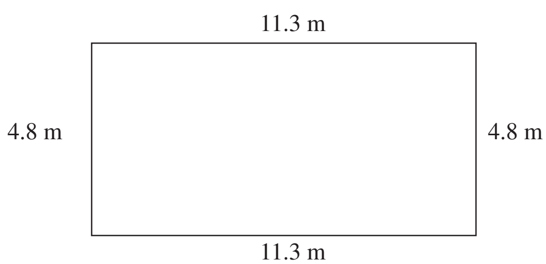
Example 2: Sammi was excited when the value of her stock went up. At the beginning of the month, the value was \$42.90 per share, but when she checked at the end of the month, the website indicated that the value had increased by \$2.73 per share. What was the per share value of her stock at the end of the month?

Solution:

To find a new value of something after an increase, and the amount of the increase to the starting value: First, even though we were not told to estimate, it's a good idea, so we round each number to the nearest dollar before adding: $43 + 3 = 46$ for an estimate of $\$46$. When we calculate the actual sum, we get: $42.90 + 2.73 = 45.63$. The new value of her stock is: \$45.63 per share. Notice we have included the units in the answer instead of just writing 45.63.

Exercise B: Bridgit was excited when her son's pediatrician told her that her son had grown a lot since their last visit. At their last visit, her son was measured at 45.5 inches, and the doctor told her that he had grown by 3.5 inches. What was her son's new height?

Example 3: The Community Gardening Society wanted to make a short fence to surround their garden. The plot is a rectangle that measures 11.3 meters by 4.8 meters. How many meters of fencing do they need to buy?



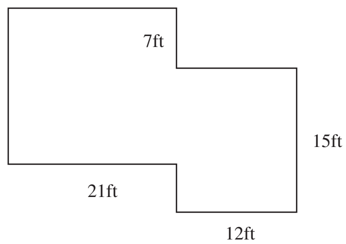
Solution 1:

The amount of fencing that they need is the total distance around the rectangle, or, what is called the *perimeter* of the rectangle. Since it's a rectangle, the side measurements are repeated on the opposite sides (see figure), so the total is $11.3 + 4.8 + 11.3 + 4.8$. Since they didn't specify a level of rounding, we will use the method outlined on page 49. 11.3 is the largest number, so we will round to the nearest ten meters. Our estimate is now $10 + 0 + 10 + 0 = 20$ meters, and with a little more work, the actual perimeter is 32.2 meters.

Solution 2:

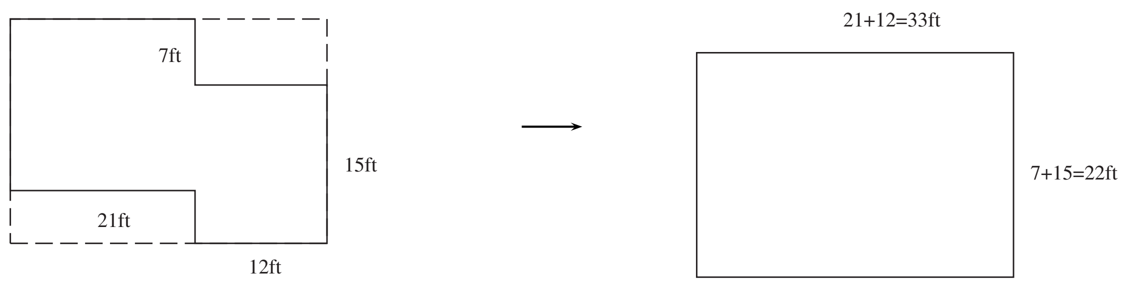
You may want a greater level of accuracy for the estimate and use technique 2. 4.8 is the smallest number, so this time we round to the nearest whole meter. Our estimate is now $11 + 5 + 11 + 5 = 32$ which isn't much more difficult to compute, but is a lot more accurate.

Example 4: Find the perimeter of the following figure:



Solution:

The Perimeter of more complicated figures can be computed with a little effort, and with what we math folks call *problem solving*. The first thing to realize, is that the perimeter of this figure is the same as the perimeter of the rectangle that it fits into. (Talk with your classmates or your instructor until you understand why this is true!)



From this picture we see that the dimensions of the surrounding rectangle are 33 feet by 22 feet, giving us our estimate of $30+20+30+20 = 100$ feet, and our actual value of $33+22+33+22 = 110$ feet.

Exercise 4.4.1 Real Life Addition Exercises

For the following exercises, write the sum that is needed to answer the question, then estimate the sum by using an estimation technique of your choice, then find the actual sum. Include appropriate units in your answers.

1. Clarence goes to the store and starts putting items in his cart. As he is approaching the checkout line, he wants to know what his total charge will be. The items in his cart are:

   sugar \$1.75, eggs \$2.50, bread \$2.25, milk \$3.19, cheese \$4.49, cereal \$3.69,
   2 boxes of Mac and Cheez at \$0.79 each, soup \$2.45, juice \$5.50, gum \$0.44.

   (a) Estimate:

   (b) Actual:

2. Hector wants to build a frame for a picture that is 8 inches tall and 10 inches wide. He plans to buy a single piece of wood and cut it up to make the frame. How long should the piece of wood be?

   (a) Estimate:

   (b) Actual:

3. As a sales rep, Marta has to drive a lot. One day, she drove 11.8 miles from San Francisco to Oakland, then she went 41.6 miles from Oakland to Napa. She had lunch in Napa, then drove 39.3 miles to San Rafael, and ended her day in San Jose, which is 36.8 miles from San Rafael. How far did she drive that day?

   (a) Estimate:

   (b) Actual:

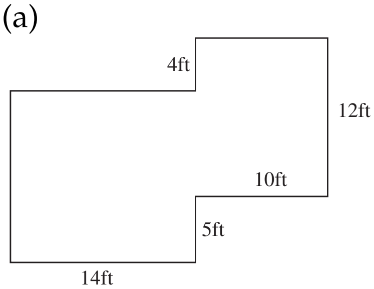
4. After finally convincing his spikey-haired boss, Dilbert was able to get a raise of $1945 per year from his old salary of $40,275 per year. What is his new salary?

   (a) Estimate:

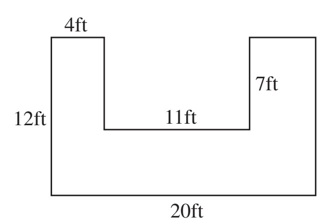
   (b) Actual:

5. Find the perimeter of each figure.

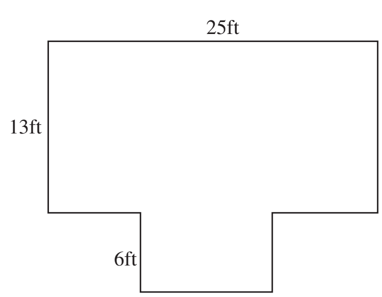
(a)



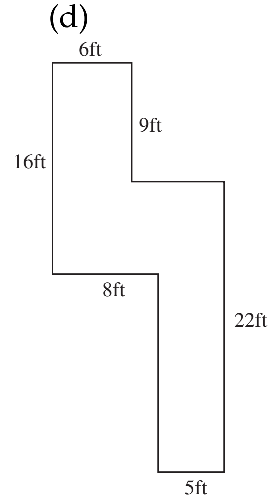
(b)



(c)



(d)



6. One month, Henry and Henrietta's bills were out of control. Their PG&E bill was $186.82, their phone bill was $62.37, their rent was the normal $1245, their credit card minimum payment was $89.79, and their Chevron charge was $183.28. What is the total of these bills?

   (a) Estimate:

   (b) Actual:

7. Measure the height and width of a door to the nearest centimeter, then calculate the door's perimeter.

   (a) How are you able to measure such a long distance in centimeters?

   (b) Height to the nearest centimeter:

   (c) Width to the nearest centimeter:

   (d) Estimate of perimeter:

   (e) Actual perimeter:

8. The number of books published about cats for various years are shown in the table below.

Year	Number of Books
1999	138
2000	98
2001	92
2002	73
2003	73
2004	120

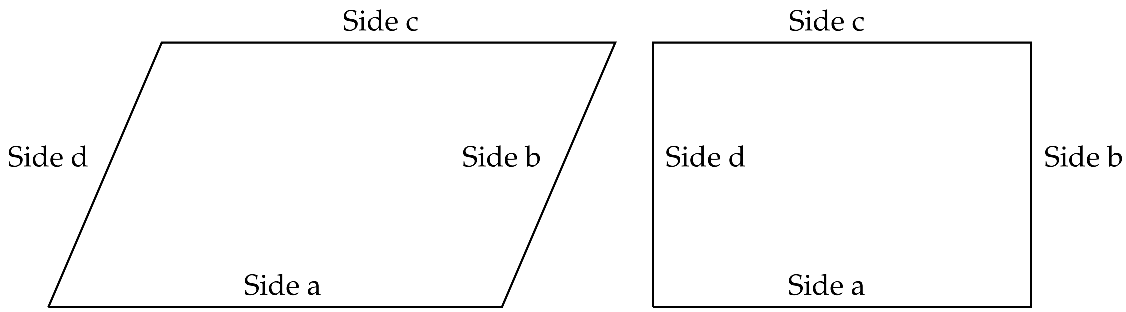
   What was the total number of books published about cats between the years 1999 and 2004?

   (a) Estimate:

   (b) Actual:

Exercise 4.4.2 Use the diagram to help answer the questions. Include appropriate units in your answers.

Joe and Kathy are neighbors who are fencing their backyards in the following shapes:



Joe's back yard   Kathy's back yard

1. Measure each side of the pictures of Joe's and Kathy's backyards in centimeters. Round each measurement to the *nearest* $\frac{1}{10}$ *centimeter*, and record it in the table. Assume that every centimeter on the picture represents one meter in real life. Record the real life distances. Be sure to write units for all of your values!

Side	Joe's picture	Joe's actual	Kathy's picture	Kathy's actual
a				
b				
c				
d				

2. How many meters of fencing does Joe need?
3. How many meters of fencing does Kathy need?
4. Who needs more fencing, Joe or Kathy?
5. How much more do they need?

Activity 4.5 Pascal's Triangle

Objective: To practice addition and pattern recognition.

Materials: Pencil.

Group size: 2 to 4.

Find a pattern in the numbers given in the triangle below. Use the pattern(s) you find to help you fill in the missing numbers. Hint: Each row's numbers come from working with the numbers in the row above it.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 ___ ___ ___ ___ 1

1 ___ ___ ___ ___ ___ 1

1 ___ ___ ___ ___ ___ ___ 1

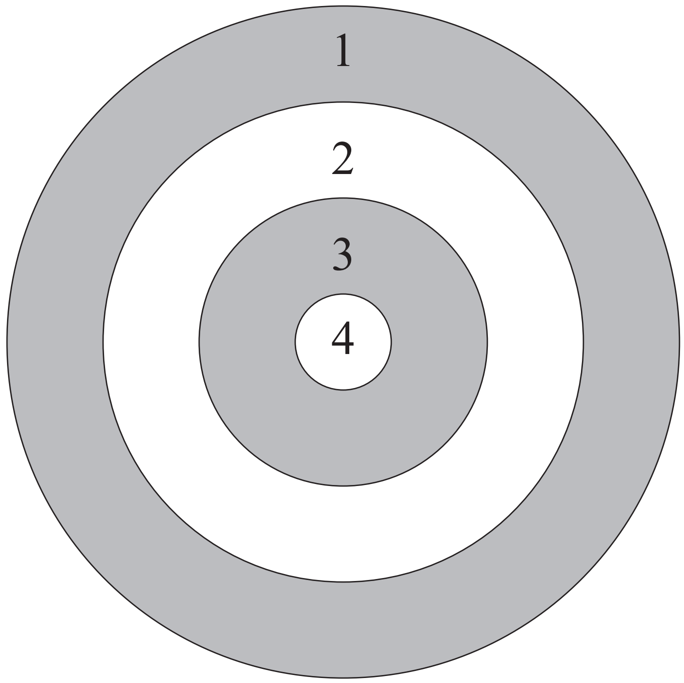
1 ___ ___ ___ ___ ___ ___ ___ 1

Activity 4.6 Darts

Objective: To practice addition, strategy, and organization skills.

Materials: Paper, pencil.

Group size: 2 to 4.



Imagine you are given four darts to throw at the dartboard shown above. The rules for playing are simple. You throw your darts at the board and add up the points you get depending on which ring your dart lands in. For now, assume you hit the board all four times.

1. What is the highest score you could get and how would you get it?

2. What is the lowest score you could get and how would you get it?

3. What are all of the possible scores?

4. Make a list of all the possible score combinations you could get playing this game. For example, you could score 11 by getting $2+2+3+4=11$ and also by scoring $4+4+2+1=11$. **Ignore** the order in which you score – treat $2+2+3+4$ the same as $4+3+2+2$ and so on.

5. From your list in (4), how many *different* ways are there to score 10? (Remember, treat $2+2+3+4$ the same as $4+3+2+2$).

6. Which score is most common (has the most ways of getting it)?

7. Suppose that it is possible to score a 0 by missing the dartboard entirely. What are all of the possible scores now?

4.5 Introduction to Fractions

We know from the way it's said, that quarters are a quarter of a dollar. But where is this term coming from? A little history of money in the U.S. will help. The following is an excerpt from http://www.collectsource.com/americas.htm:

> "The Spanish Dollar quickly became the most popular coin in North America. It is even thought by some that if Washington did throw a coin across the Potomac, it was likely to have been America's First Silver Dollar. Every day commerce was lubricated by this remarkable coin, and the terminology which developed by using it became so deeply embedded in American culture that it remains with us to this day.
> **Pieces of Eight**
>
> Because America's First Silver Dollar was often cut into eight pie shaped 'bits' in order to make change, the intact coin became known as a 'Piece of Eight.' Since the entire Piece of Eight had a value of 8 Reales, each bit was valued at one eighth of the total. Two bits equaled a quarter, four bits a half dollar and six bits three quarters of a dollar. Did you ever spend two bits?—Then you were living the legacy of America's First Silver Dollar! To put the value of America's First Silver Dollar into perspective, an average worker during the colonial era earned about 2 bits a week!"

The following pictures are of coins of this time.

The following activity simulates history in order to introduce fractions and adding fractions.

Activity 4.7 Shopping with Fractions

Objective: To introduce the idea of fractions using money.

Materials: 8 breakable silver dollars.

Group Size: 3 to 4

Set up: Choose one person in the group to be the merchant, the others to be customers.

- Customer one starts with 2 dollars.
- Customer two starts with 3 dollars.
- Customer three (if there are that many in your group) starts with 3 dollars.

Procedure: Act out the following scenario, using the coins as props. Answer each question, using the coins and your discussion to help you.

Use the following prices to complete the activity:

Pigs: 2 for a dollar
Chickens: 4 for a dollar
Coffee: 8 pounds for a dollar

The first customer comes in to buy one chicken.

1. How much is the total charge for customer one?
   (a) Write as a fraction of a dollar.
   (b) Write in standard decimal notation.

   Have customer one pay the merchant for the goods.

2. How can the merchant give change?

3. After making the purchase, how many full coins does customer one have left? How many "quarters"?

   Now, customer two buys one pig and one chicken.

4. How much is the total charge for customer two?
   (a) Write as a fraction of a dollar.
   (b) Write in standard decimal notation.

   Have customer two pay the merchant for the goods.

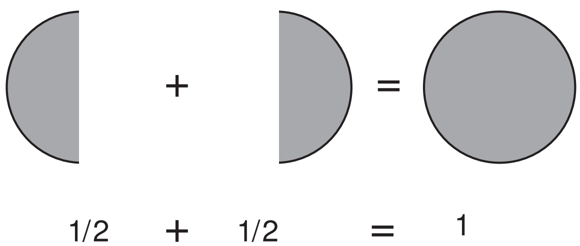
5. How many full coins does customer number two have left? How many "quarters"?

Customer three buys 9 chickens and four pounds of coffee.

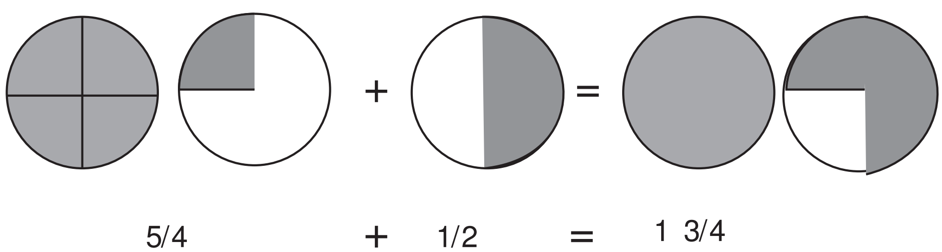
6. How much is the total charge for customer three?

Have customer three pay the merchant for the goods.

7. How many full coins does customer number three have left? How many "quarters"?
8. If the three customers combine their change into one pile, how many full coins do they have altogether? How many quarters?
9. How much money do they have altogether?
10. Write an addition problem, using decimal notation that models the total that you just computed.
11. Now, re-write the sum using fraction notation, that is write $\frac{1}{4}$ for one quarter, $\frac{2}{4}$ for two quarters, etc.
12. Is there more than one way to write the total in fraction form? If so, write the total in as many correct ways as you can think of (up to five).
13. Compute the following sums. Use your coins to help. Draw a picture of the problems using the coins. For example, if the problem was $\frac{1}{2} + \frac{1}{2}$, you would see that the answer is 1. Then, you would draw a picture similar to the one below.



The sum $\frac{5}{4} + \frac{1}{2}$ is illustrated here:



(a) $\frac{1}{2} + \frac{1}{4}$

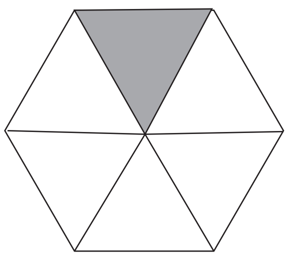
(b) $\frac{3}{2} + \frac{3}{4}$

(c) $\frac{1}{4} + 1\frac{1}{2}$

4.6 Fractions

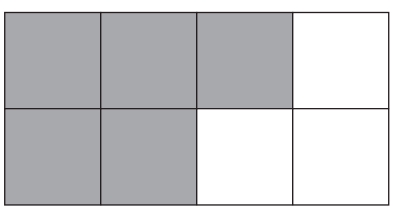
A fraction represents a part of a whole, like a dime is a part of a dollar or a piece of pie is part of the whole pie. The bottom number in a fraction, the *denominator*, tells us the number of pieces the whole was divided into, so the 8 in $\frac{3}{8}$ of a pizza means the whole pizza was cut into 8 pieces. It gives some sense of the size of the pieces, the larger the number, the more pieces, so the smaller each individual piece must be. The top number, the *numerator*, describes the number of pieces we have out of the total. The 3 in $\frac{3}{8}$ of a pizza means we have three slices.

Example 1:



Solution: Since the shape can be broken into 6 triangles of the same size and one of them is shaded, we say that one-sixth (written $\frac{1}{6}$) of the shape is shaded.

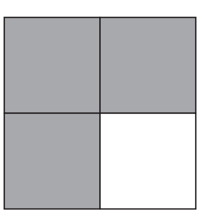
Example 2:



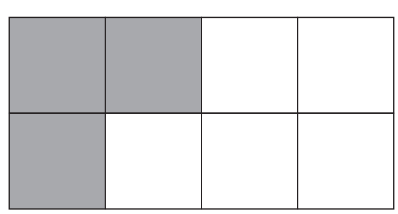
Solution: Since the shape can be broken into 8 squares of the same size and five of them is shaded, we say that five-eighths (written $\frac{5}{8}$) of the shape is shaded.

Exercise 4.6.1 State what fraction of the following shapes is shaded.

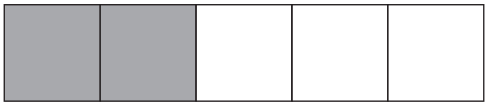
1. What fraction is shaded?



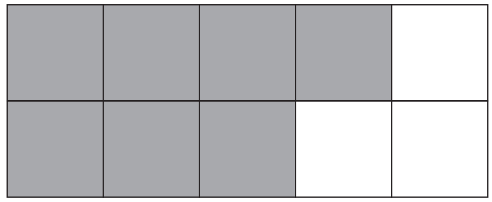
2. What fraction is shaded?



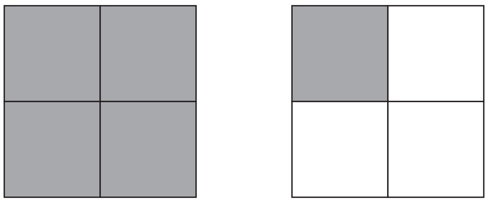
3. What fraction is shaded?



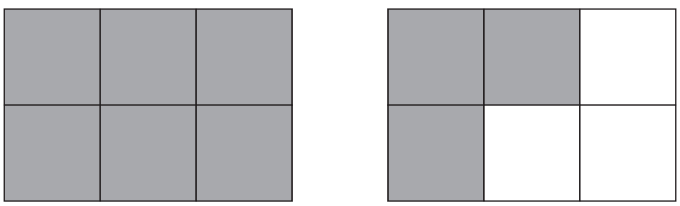
4. What fraction is shaded?



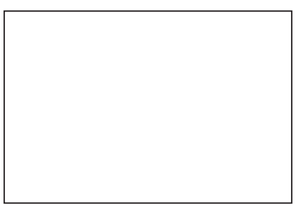
5. What fraction is shaded? (Hint: Since more than one whole rectangle is shaded, the number of pieces shaded (the numerator) will be greater than the number of pieces in one whole (the denominator). A fraction in which the numerator is greater than the denominator is called an ***improper fraction***.)



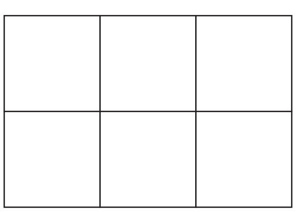
6. What fraction is shaded?



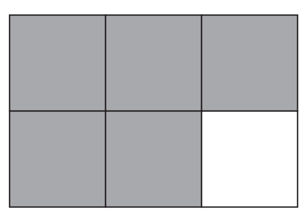
Example: Shade $\frac{5}{6}$ of the rectangle.



Solution: Break the rectangle into six pieces ***of equal size***,

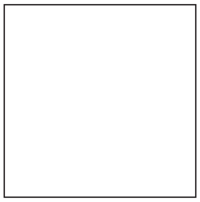


then shade five of them.

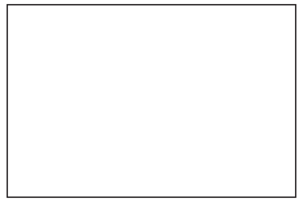


Exercise 4.6.2 Shade the fraction of the shape indicated.

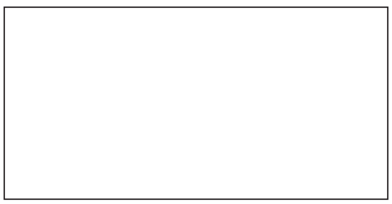
1. Shade $\frac{1}{4}$ of the rectangle.



2. Shade $\frac{1}{6}$ of the rectangle.

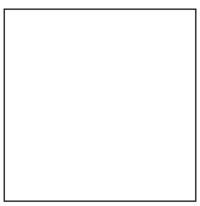


3. Shade $\frac{7}{8}$ of the rectangle.

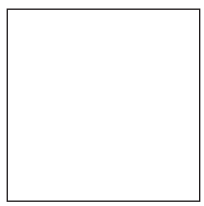


4. Shade $\frac{3}{5}$ of the rectangle.

5. Shade $\frac{2}{4}$ of the rectangle.

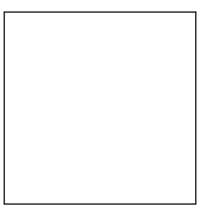


6. Shade $\frac{1}{2}$ of the rectangle.

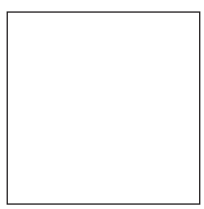


7. Describe any similarities and differences between problems 5 and 6.

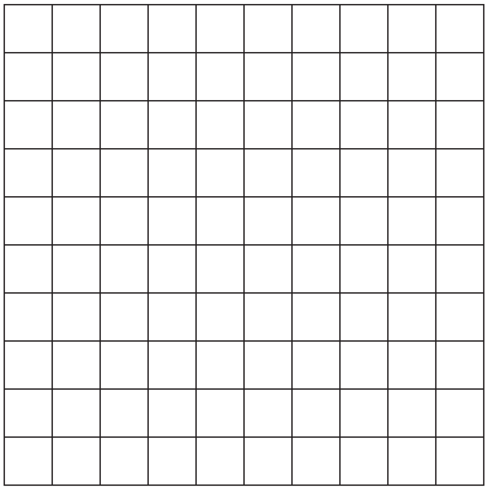
8. Shade $\frac{7}{4}$ of the rectangle.



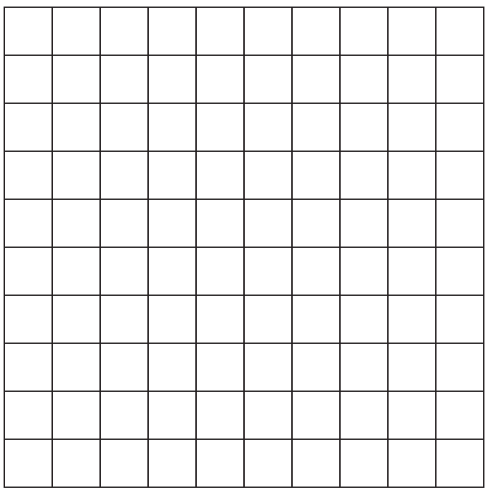
9. Shade $\frac{5}{2}$ of the rectangle.



10. Shade $\frac{13}{100}$ of the rectangle. Another way to ask this same question is to say, "Shade 13% of the rectangle", since "percent" means "out of 100".



11. Shade $\frac{29}{100}$, that is, shade 29% of the rectangle.



12. What fraction of a dollar is 80 cents?

13. What fraction of a dollar is 20 cents?

14. What fraction of a dollar is $1.50?

15. What fraction of an hour is 30 minutes?

16. What fraction of an hour is 20 minutes?

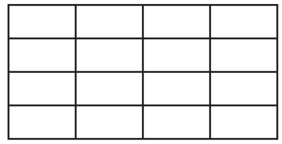
17. What fraction of an hour is 7 minutes?

18. What fraction of an hour is 90 minutes?

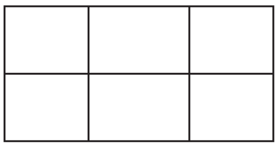
19. Shade $\frac{1}{2}$ of the given rectangles.

a.

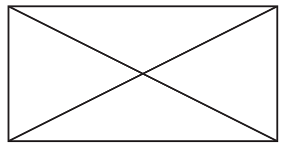
b.



c.

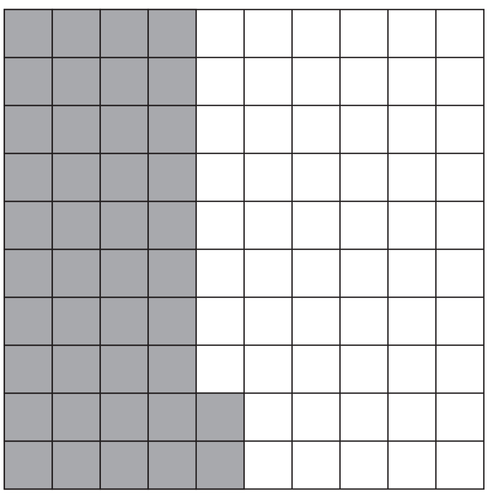


d.

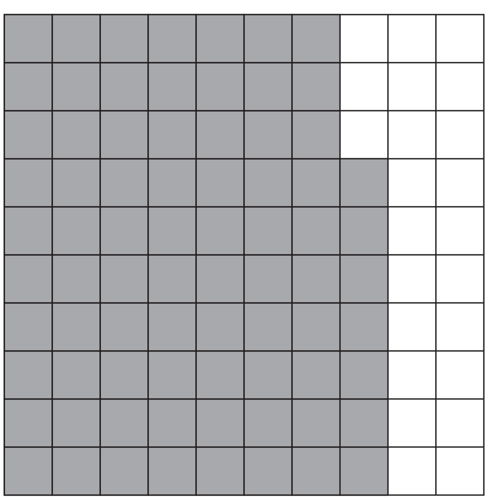


Exercise 4.6.3 State what percent of the following rectangles is shaded. Remember, the percent is just the number of pieces out of 100.

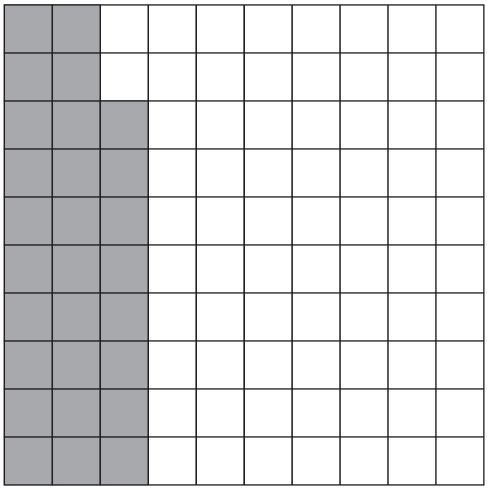
1. What percent of the following is shaded?



2. What percent of the following is shaded?



3. What percent of the following is shaded?



Exercise 4.6.4 Fill in the table with the missing hundredths ($\frac{1}{100}$s, 0.01s, or %s). Recall that percents are the same as hundredths. That is, 1 percent equals 0.01 equals $\frac{1}{100}$. The three are just different representations of the same amount.

No.	Fraction Representation	Decimal Representation	Percent Representation
1.	$\frac{17}{100}$	0.17	17%
2.	$\frac{3}{100}$	0.03	3%
3.	$\frac{150}{100}$	1.5	150%
4.	$\frac{28}{100}$		28%
5.		0.7	70%
6.	$\frac{20}{100}$	0.2	
7.	$\frac{38}{100}$		
8.		0.95	
9.			63%
10.		0.8	
11.		0.23	
12.	$\frac{122}{100}$		
13.		0.09	
14.			780%
15.		13	1300%

4.7 Measuring Lengths with Inches

Activity 4.8 Measuring with Fractions

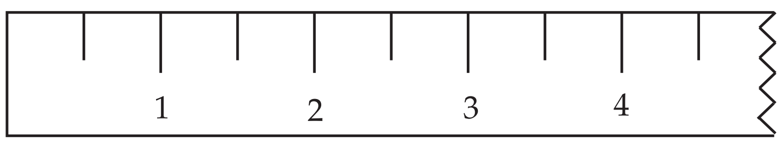
Objective: To further investigate fractions while introducing measuring with inches.

Materials: Long, thin rectangle made of card stock.

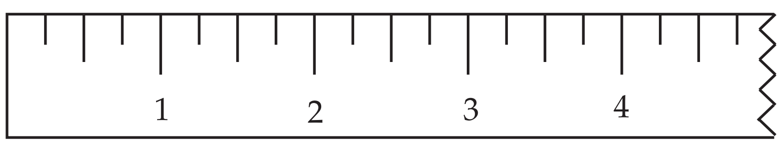
Procedure: In the following activity, you will create your own unit of measure, make a ruler that measures in that unit, then measure the length of given lines, accurate to the nearest quarter unit.

1. Choose a part of your body that is approximately one inch long. Use this length as your "unit" of measure, and mark your blank ruler with whole unit markings up to the end of the rectangle. Your ruler would look something like this:

2. With slightly shorter lines, mark the half way point between each unit. At this stage your ruler would look something like this:



3. With still shorter lines, mark the one-quarter and three-quarters points between each unit. At this stage, your ruler would look something like this:



4. Using your new ruler, make an accurate drawing of the ruler including all of the markings.

5. Using your new ruler, measure the lengths of the following lines, accurate to the nearest quarter of a unit. Compare your answers with other people in the class. Discuss how/why someone could get a different answer for any of the measurements.

   (a)

   (b)

   (c)

   (d)

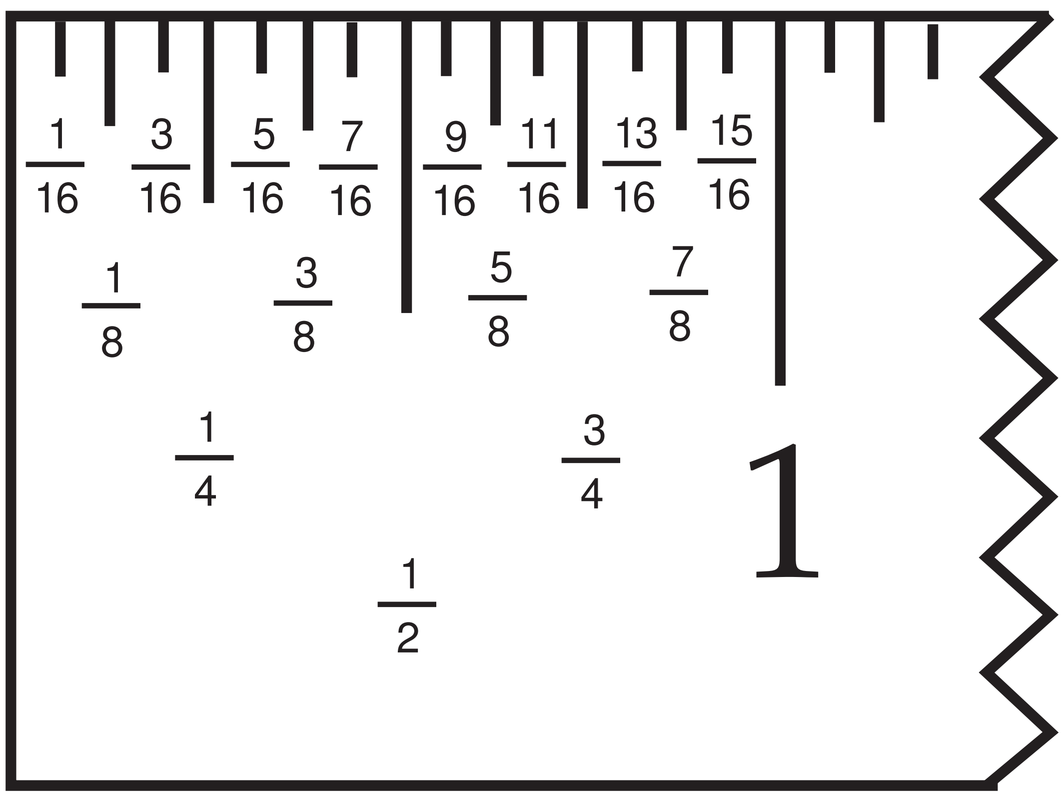
   (e)

What parts make up an inch?

Up until the previous activity, we have been measuring distances using centimeters. Centimeters are part of the *metric* system of measurement, so they are based on decimals. When you need a finer measurement, you use tenths of a centimeter, or hundredths of a centimeter.

Inches are part of the *English* system. Instead of breaking things into tenths or hundredths (powers of 10), inches are broken up into halves, quarters, eighths, and sixteenths. The denominators of these fractions are all powers of 2 since each part of a fraction is half the size of the next bigger part.

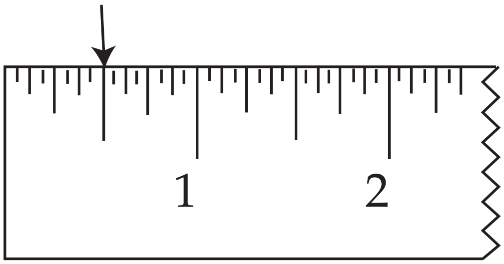
The following is an enlarged picture of an inch. Notice the markings that show the fractions of an inch.



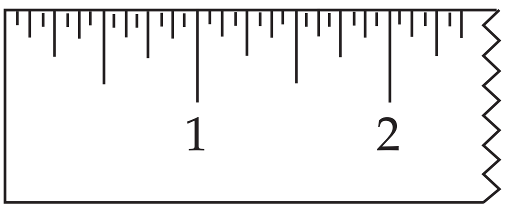
Exercise 4.7.1 Mark the following lengths on the given rulers.

Example: $\frac{1}{2}''$

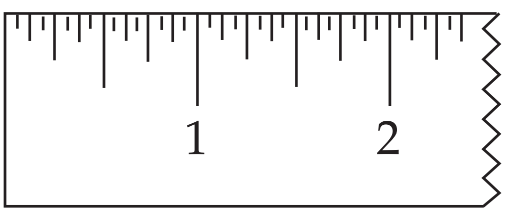
Solution:



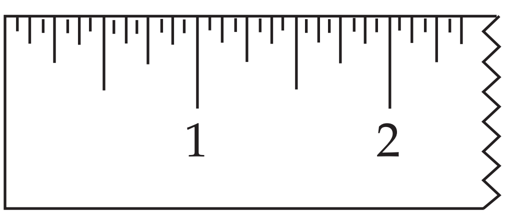
1. $\frac{1}{4}''$



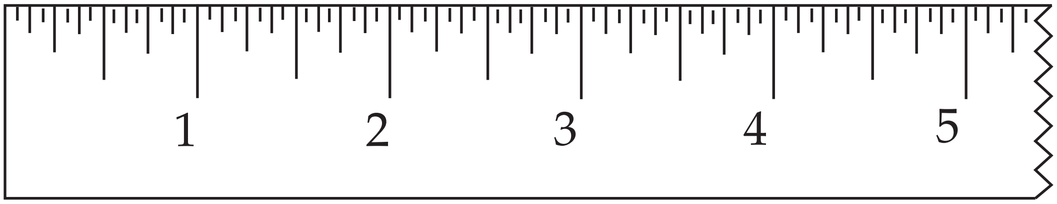
2. $\frac{3}{4}''$



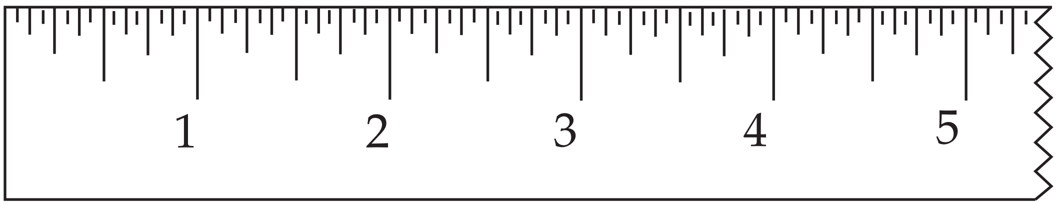
3. $1\frac{3}{8}''$



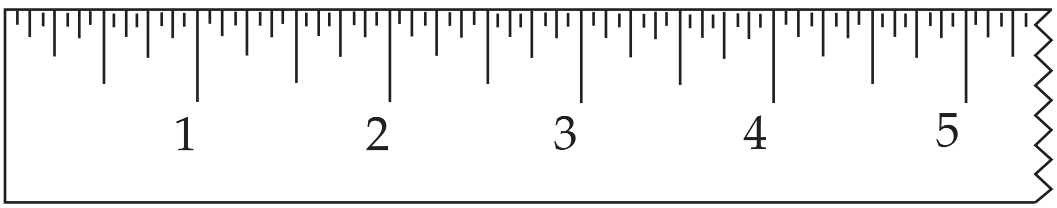
4. $3\frac{5}{8}''$



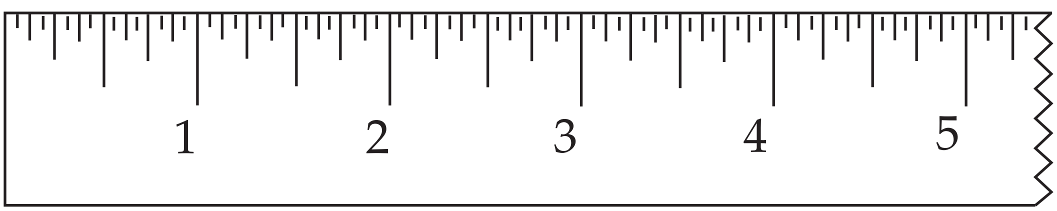
5. $2\frac{1}{16}''$



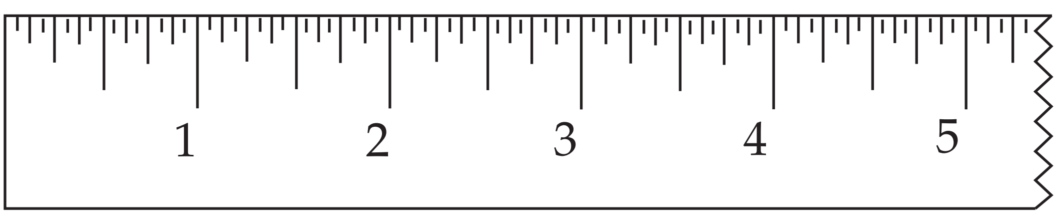
6. $3\frac{1}{4}''$



7. $4\frac{3}{16}''$



8. $2\frac{7}{8}''$



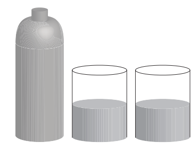
Home Project: Fraction Bottle

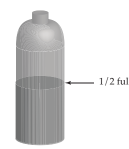
A fraction represents a part of a whole, like a dime is a part of a dollar or a piece of pie is part of the whole pie. The most common fraction, $\frac{1}{2}$, is the source of almost all the fractions you will need in daily life. The purpose of this assignment is to find and mark some common fractions on a plastic bottle.

Materials: You will need a plastic bottle, like the ones used for water or soda (any size will do).

You will also need two identical containers, like drinking glasses or more plastic bottles.

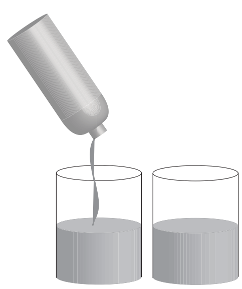
You will also need a Sharpie to write on the bottle.



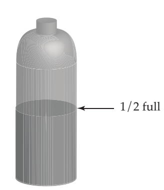


Procedure: We want to find the half full point on the bottle (see right).

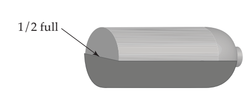
1. In order to do this begin by filling the bottle with water (to the top).
2. Pour the water out into the two identical containers being careful to put an equal amount in each glass. (figure 1).



3. Now throw out the water in one glass and pour the contents of the other back into the bottle.
4. Put a mark on the side of the bottle showing the water level. This is the half full point.

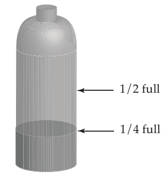


5. Now turn the bottle sideways and mark the half full point on the bottom of the bottle.



Now we want to mark the point where half of the remaining water reaches (half of a half).

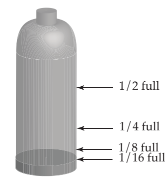
6. Using the water remaining in the bottle, repeat the procedure above by distributing it equally between the two containers.



7. Empty the water in one of the containers and pour the remaining water back in the bottle.

8. Mark both the side and the bottom of the bottle showing the the quarter-full points.

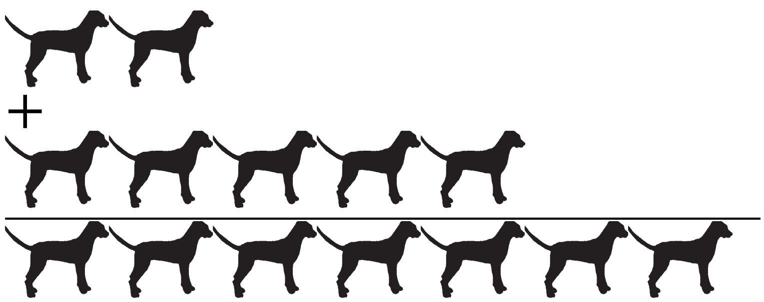
9. Repeat the process above until you have made the four marks for $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$. on both the side and the bottom of the bottle.



4.8 Adding Fractions

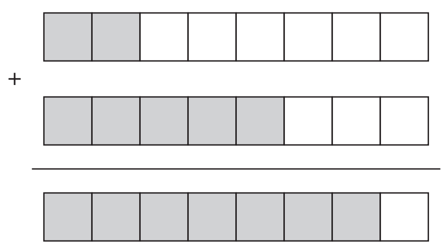
As long as the pieces are the same size, adding fractions is no different than adding any other items that are the same. Two dogs plus five more dogs adds up to seven dogs just like two eighths plus five more eighths adds up to seven eighths. In pictures this looks like:

Adding Dogs:



Adding Eighths:

$$\begin{array}{r} \frac{2}{8} \\ + \quad \frac{5}{8} \\ \hline \frac{7}{8} \end{array}$$



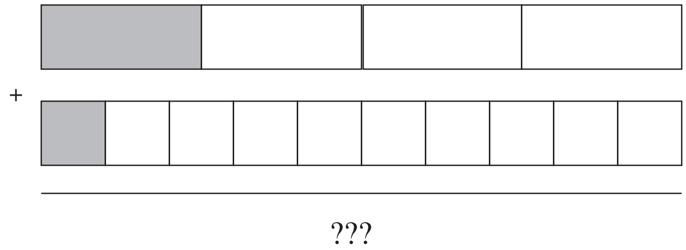
Adding Dimes:

Or, using fraction or decimal notation since a dime is just $\frac{1}{10} = 0.1$ of a dollar,

$\frac{2}{10} + \frac{5}{10} = \frac{7}{10}$ or $0.2 + 0.5 = 0.7$

We have problems when we try to add a quarter to a dime:

$$\begin{array}{r} \frac{1}{4} \\ + \quad \frac{1}{10} \\ \hline \frac{2}{?}? \end{array}$$

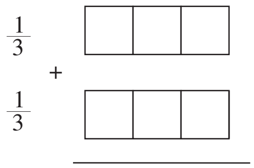


But we know how to add \$0.25 to \$0.10! We know the total is \$0.35! In fraction notation, since 1 cent is one hundredth of a dollar, this is the same as saying,

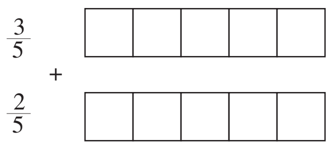
English	Fractions	Decimals
One quarter and one dime	$\frac{1}{4} + \frac{1}{10}$	
equals 25 cents and 10 cents	$= \frac{25}{100} + \frac{10}{100}$	$= 0.25 + 0.10$
equals 35 cents.	$= \frac{35}{100}$	$= 0.35$

Exercise 4.8.1 Shade the boxes to show the given fraction addition, then draw and shade a box (or boxes) to represent the sum. When you are done shading, write the sum in fraction form. Be sure to make each whole strip the same size, as well as making each piece within the whole the same size. You may need more than one whole for some of the answers.

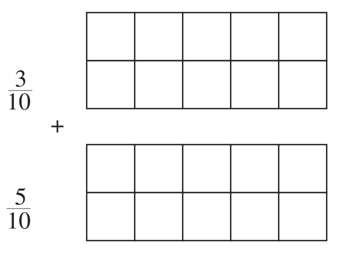
1. $\frac{1}{3} + \frac{1}{3}$



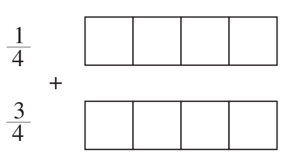
2. $\frac{3}{5} + \frac{2}{5}$



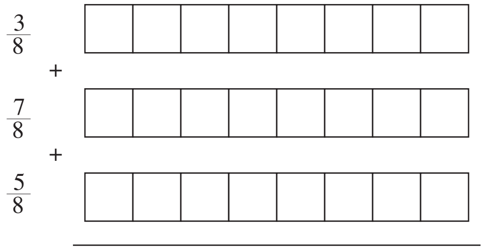
3. $\frac{3}{10} + \frac{5}{10}$



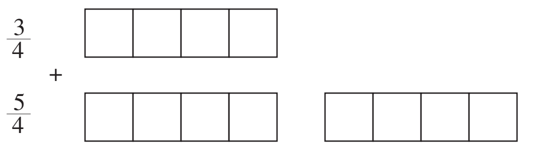
4. $\frac{1}{4} + \frac{3}{4}$



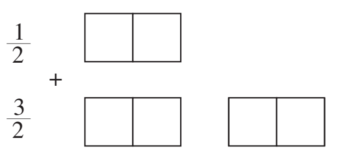
5. $\frac{3}{8} + \frac{7}{8} + \frac{5}{8}$



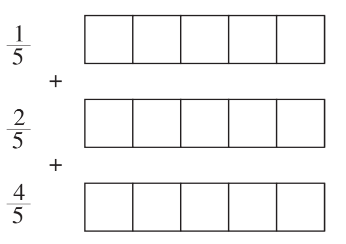
6. $\frac{3}{4} + \frac{5}{4}$



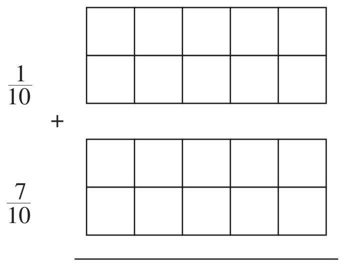
7. $\frac{1}{2} + \frac{3}{2}$



8. $\frac{1}{5} + \frac{2}{5} + \frac{4}{5}$

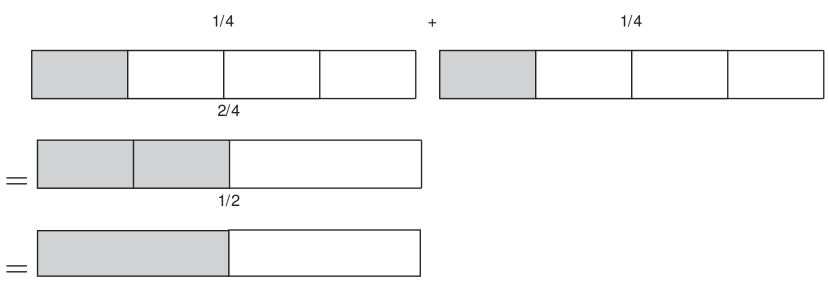


9. $\frac{1}{10} + \frac{7}{10}$



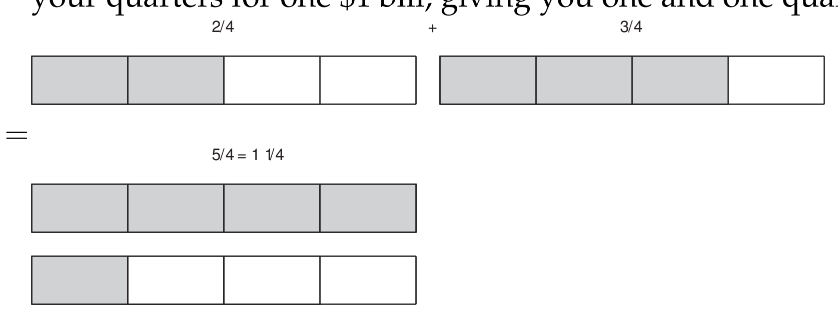
4.9 Equivalent Fractions

When you add money, sometimes you can exchange many smaller bills with a larger bill if you have enough of the smaller bill. The same is true for coins, the fractions that the coins represent, and fractions in general. For example, if you add 1 quarter to 1 quarter you will get two quarters. You may exchange your quarters for one half dollar.



$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Similarly, if you add 2 quarters to 3 quarters you will get 5 quarters. You may exchange four of your quarters for one $1 bill, giving you one and one quarter dollars.



$\frac{2}{4} + \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}$

Equivalent fractions represent equal amounts. For example, suppose you wanted to spend $\frac{1}{2}$ of the $24 in your wallet. You could split the $24 into two equal $12 pieces and spend one of the two pieces. Equivalently, you could split the $24 into four equal $6 pieces and spend two of the four pieces. The first time, you are spending $\frac{1}{2}$ of your money, while the second time you are spending $\frac{2}{4}$. Both times, however, you spend $12! The list of fractions equivalent to $\frac{1}{2}$ is literally endless.

That is: $\frac{1}{2} = \frac{2}{4} = \frac{12}{24} = ...$

$\frac{1}{2}$ is said to be in *simplest form* because of all of these fractions that represent the same amount, $\frac{1}{2}$ has the smallest denominator.

Note: A fraction in which the numerator is greater than the denominator is called an *Improper Fraction*, and they can be written in an equivalent form called a *Mixed Number*. For example, from above we saw that one and a quarter is equivalent to five quarters. $1\frac{1}{4}$ is a mixed number, while $\frac{5}{4}$ is an improper fraction.

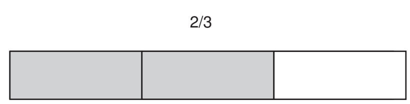
If a fraction has a numerator that is smaller than the denominator, it is called a *Proper Fraction*. $\frac{3}{4}$ is an example of a proper fraction. Proper fractions don't have a mixed number equivalent.

We can use pictures to show that two fractions are equivalent. We just have to make sure that the size of each rectangle is the same, and that the total size of the shaded parts are the same. Here are two examples:

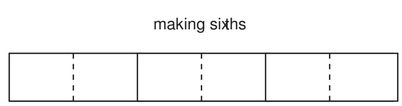
Example: $\frac{2}{3}$

Solution 1:

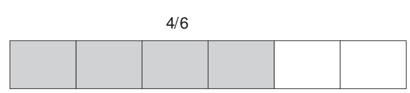
$\frac{2}{3}$



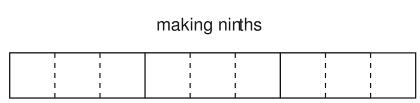
add lines to make sixths



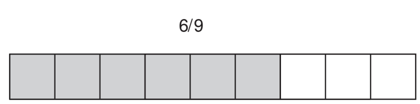
$\frac{4}{6}$



add lines to make ninths

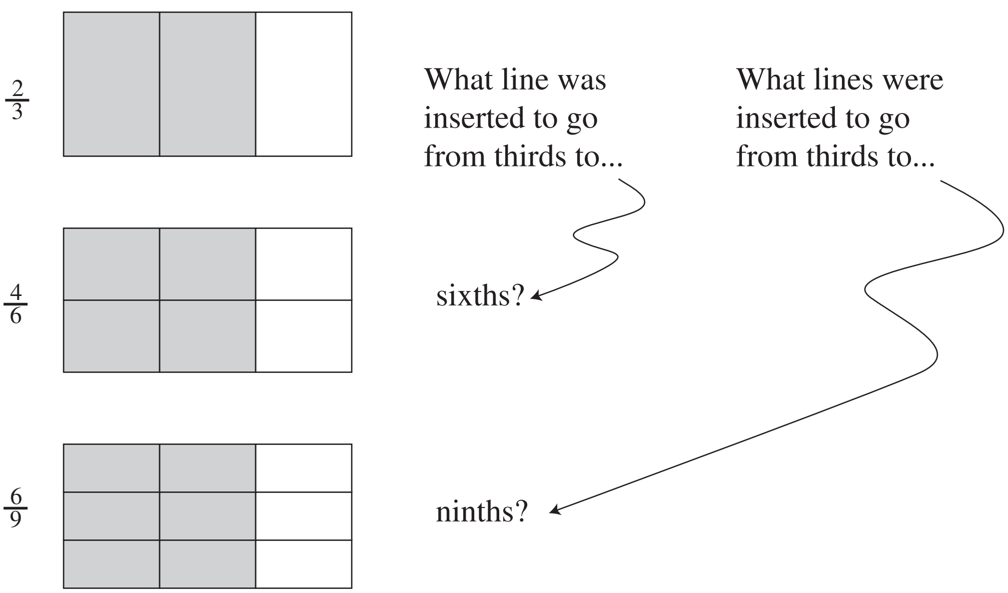


$\frac{6}{9}$



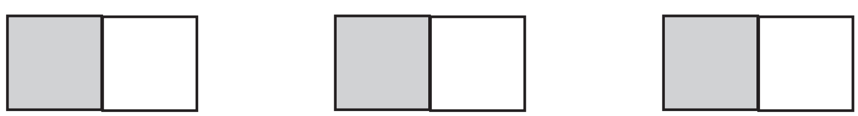
Here is another way to show the same fractions equivalent to $\frac{2}{3}$:

Solution 2:

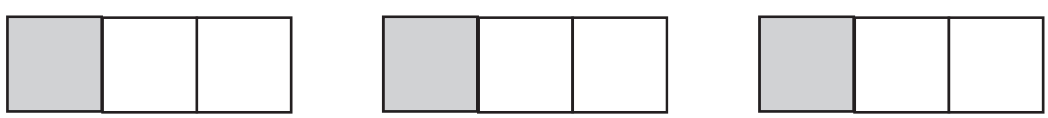


Exercise 4.9.1 Write two other fractions, with different denominators, that are equivalent to the given fraction. For the given fraction and each of your equivalent fractions, draw a shaded rectangle (or more than one if the given fraction is an improper fraction) showing that they all represent the same amount. For the first two problems, the given fraction has been drawn three times. Leave the first one alone, then draw lines on the other two to make equivalent fractions. For problems 3-10, draw each picture completely.

1. $\frac{1}{2}$



2. $\frac{1}{3}$



3. $\frac{2}{5}$

4. $\frac{3}{4}$

5. $\frac{3}{2}$

6. $\frac{1}{8}$

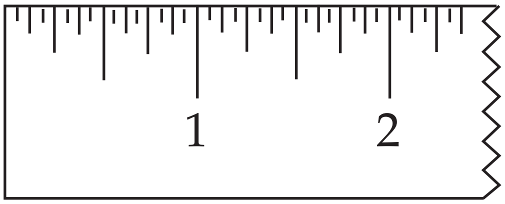
7. $\frac{3}{8}$

8. $\frac{4}{3}$

9. $\frac{7}{4}$

Exercise 4.9.2 Complete the following equivalence problems. Include appropriate units in your answers.

1. Write three lengths with different denominators equivalent to $\frac{1}{2}''$.

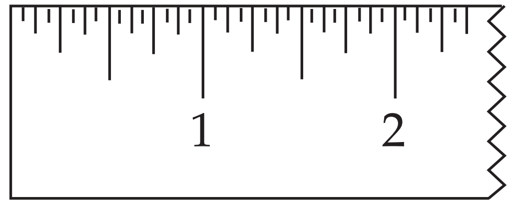


(a) ____________

(b) ____________

(c) ____________

2. Write three lengths with different denominators equivalent to $\frac{6}{4}''$.

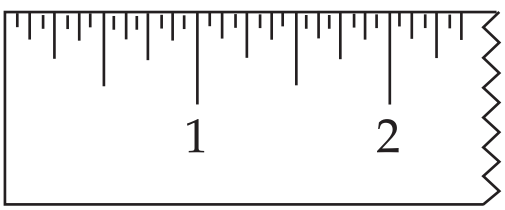


(a) ____________

(b) ____________

(c) ____________

3. Write three lengths with different denominators equivalent to $\frac{3}{4}''$.

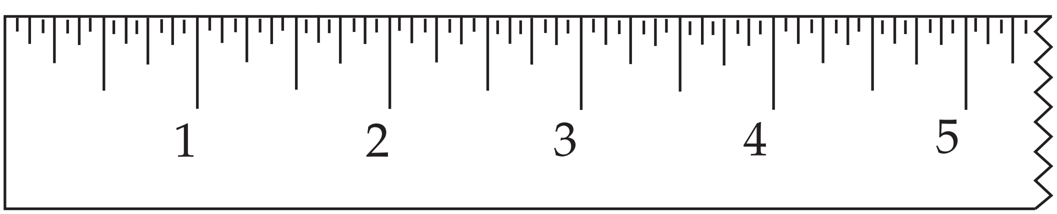


(a) ____________

(b) ____________

(c) ____________

4. Write three lengths with different denominators equivalent to $3\frac{2}{4}''$.



(a) ____________

(b) ____________

(c) ____________

4.10 Adding Mixed Numbers:

Adding mixed numbers is similar to adding whole numbers. The only difference is that the fraction parts of the numbers must be thought of as their own place value.

Example 1: Add $14\frac{3}{5} + 28\frac{4}{5}$

Solution: First estimate as usual: $10 + 30 = 40$ so that we know that the sum is approximately 40. For the actual sum, line up the tens, ones, and fifths for each number:

$$\begin{array}{r} 14\,\frac{3}{5} \\ 28\,\frac{4}{5} \\ \hline \end{array}$$

Next, add up the fifths. Since $\frac{7}{5}$ is bigger than one-whole, we break it up into $\frac{5}{5} = 1$ and $\frac{2}{5}$. The extra one is "carried" over into the one's column:

$$\begin{array}{r} {}^{1} \\ 14\,\frac{3}{5} \\ 28\,\frac{4}{5} \\ \hline \frac{2}{5} \end{array} \quad \Big\} \; \frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\,\frac{2}{5}$$

Now, with the carry, there is a total of 13 ones. As usual, we carry 10 of them as an extra ten:

$$\begin{array}{r} {}^{1\,1} \\ 14\,\frac{3}{5} \\ 28\,\frac{4}{5} \\ \hline 43\,\frac{2}{5} \end{array}$$

The final answer is the mixed number $43\frac{2}{5}$.

Example 2: Kreg was making a shopping list before baking. He was going to make 2 batches of cookies, and some banana bread. Each batch of cookies called for $2\frac{1}{4}$ cups of flour, and the banana bread called for $1\frac{3}{4}$ cups. How much flour did he need altogether?

Solution: The total is the sum of $2\frac{1}{4} + 2\frac{1}{4} + 1\frac{3}{4}$. Following the process outlined, the work should look something like this:

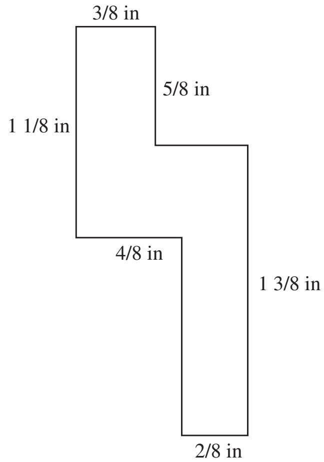
$$\begin{array}{r} {}^{1} \\ 2\,\frac{1}{4} \\ 2\,\frac{1}{4} \\ 1\,\frac{3}{4} \\ \hline 6\,\frac{1}{4} \end{array}$$

The answer is $6\frac{1}{4}$ cups.

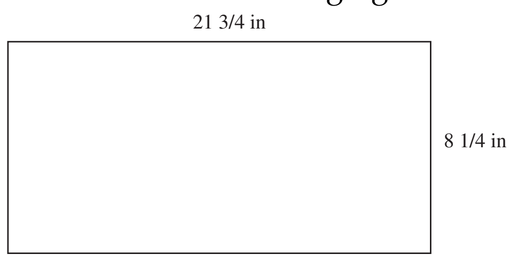
Exercise 4.10.1 Add the mixed numbers in the sums or solve the word problems. Write the answers as mixed numbers, with the fraction part in simplest form.

1. $2\frac{5}{8} + 7\frac{5}{8}$
2. $5\frac{3}{4} + 14\frac{3}{4} + 5\frac{3}{4} + 14\frac{3}{4}$
3. $1\frac{9}{16} + 4\frac{5}{16}$
4. $2\frac{3}{4} + 3\frac{1}{2}$
5. For the four day weekend, you decide to do some baking. Your chocolate chip cookie recipe calls for $1\frac{1}{4}$ cups of sugar. The banana bread needs $\frac{3}{4}$ cups of sugar. The cake you are making from scratch calls for $2\frac{3}{4}$ cups of sugar. If you make a double batch of cookies, and three batches of banana bread, and one cake, how much sugar will you need?

6. Find the perimeter of the following figure:



7. Find the perimeter of the following figure:



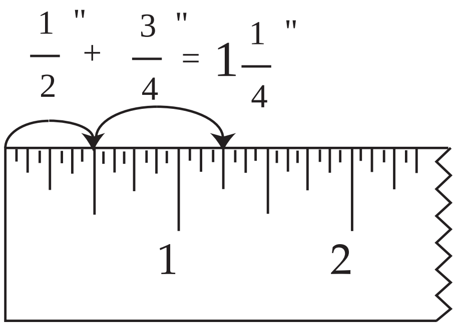
8. Count by $\frac{1}{2}$'s up to 5. Write each fraction in simplest form.
9. Count by $\frac{1}{4}$'s up to 2. Write each fraction in simplest form.
10. Count by $\frac{1}{16}$'s up to $1\frac{1}{2}$. Write each fraction in simplest form.

Exercise 4.10.2

1. In each problem, add the fractions of an inch by jumping along the ruler below.

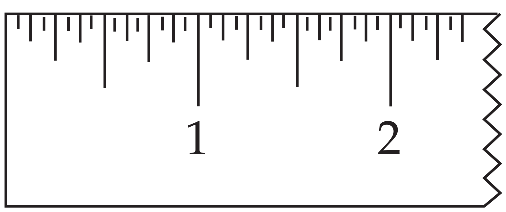
Example: $\frac{1}{2}'' + \frac{3}{4}'' =$

Solution: Start at the beginning of the ruler and "jump" a length of $\frac{1}{2}''$. Then continue on and jump an additional $\frac{3}{4}''$. The total is where you land!

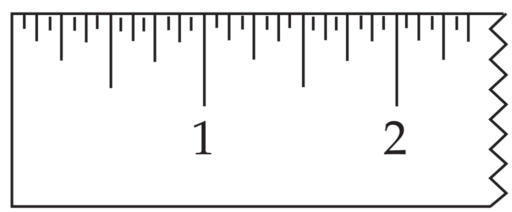


The answer is $1\frac{1}{4}$ inches.

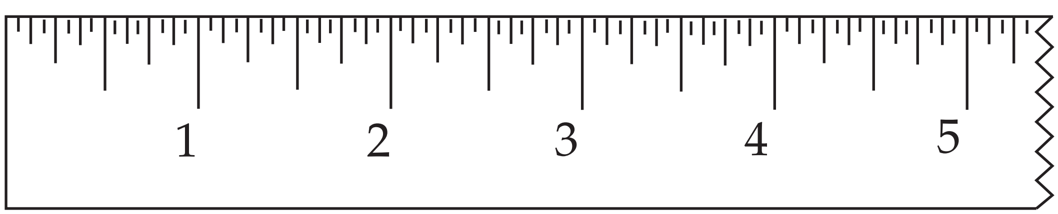
(a) $\frac{5}{8}'' + \frac{7}{8}'' =$ __________



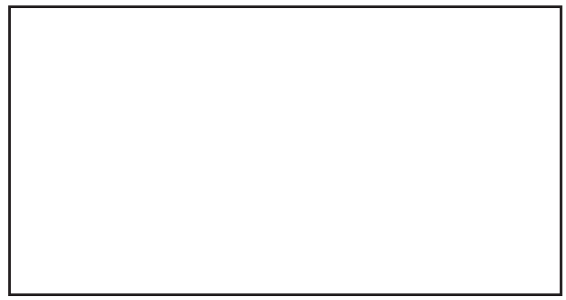
(b) $\frac{7}{16}'' + \frac{13}{16}'' =$ __________



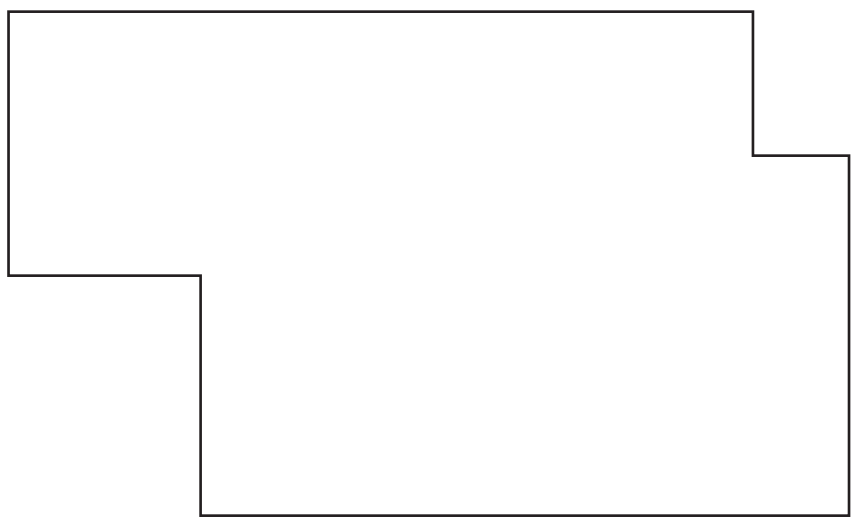
(c) $2\frac{1}{2}'' + 1\frac{3}{4}'' =$ __________



2. Using a ruler to measure the sides to the nearest eighth of an inch, confirm that the perimeter of the rectangle below is $8\frac{3}{4}$ inches. Mark the measurements on the figure.



3. Use a ruler and measure the sides of the shape below to the nearest eighth of an inch in order to determine its perimeter. Mark the measurements that you use on the figure. (Hint: You may not need to measure every side!)



Exercise **4.10.3** Add the following using any method. Recall that percents are the same as hundredths. That is, 1 percent equals 0.01 equals $\frac{1}{100}$. The three are just different representations of the same amount.

1. $40\% + 20\%$
2. $\frac{17}{100} + \frac{13}{100}$
3. $0.39 + 0.52$
4. $28\% + \frac{4}{100}$
5. $0.23 + 14\%$
6. $\frac{37}{100} + 0.52$
7. $\frac{7}{100} + 9\% + 0.05$
8. $\frac{96}{100} + \frac{38}{100}$
9. $47\% + 79\%$
10. $0.82 + \frac{61}{100}$
11. $0.3 + \frac{5}{10} + 10\%$

Activity **4.9** Fraction Paths

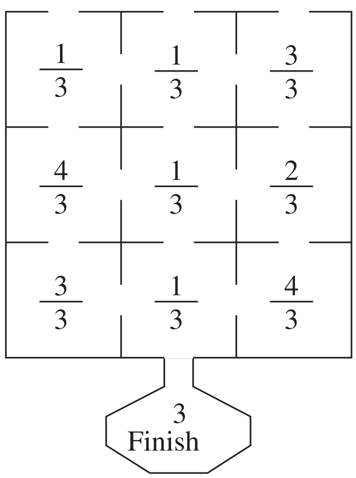
Objective: To practice adding fractions.

Materials: Pencil.

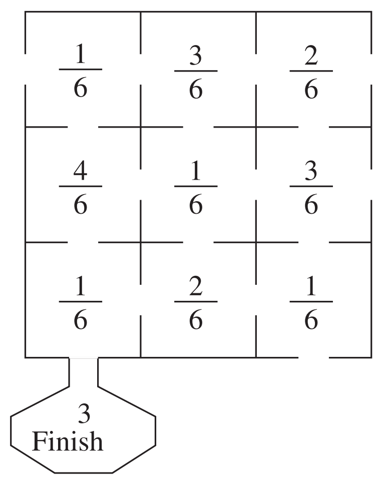
Group size: 2 to 4.

Start at any of the openings along the outside of the square and draw a path through the fractions that adds up to the number at the Finish.

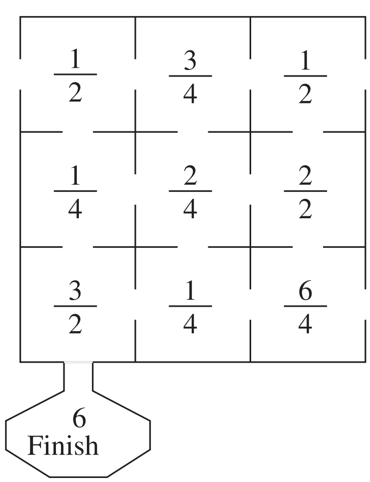
1.



2.



3.



4.