Fundamentals of Math
An Activities and Applications Approach

by Soodi Zamani, Jon Freedman, and Rick Hough

August 2010

Contents

HOW TO BE SUCCESSFUL IN A MATH CLASS

(regardless of your past experiences)

Success isn't a spectator sport. You succeed because you commit to your decision to do something and you act on that decision. People don't succeed because they're lucky, they succeed because they do something about it. You may have done well in math classes in the past or you may have had some difficulty but in the end the message is the same. If you want to learn math, if you want to pass this class, then you have to make it a conscious decision and follow a plan to see it though.

Although there are many reasons you may not have met your goals, there are also multiple strategies for succeeding in your next math class, especially if you begin with some good habits you maintain throughout the semester. From the beginning of the class, almost every day, through to the end of the semester, a successful student will follow most or all of the following:

- **Come to class.** This seems obvious, but it's critical. If you could do the work without attending class, you wouldn't be taking the class in the first place.
- **Do all the assigned work.** If you ever do get behind, get help immediately so that you can catch up with the current topics of each class.
- **If any of the material is unclear, get help from your instructor, a tutor, or a classmate.** The longer you wait to do this, the more the amount of unclear material builds up until virtually nothing makes sense in class anymore.
- **Study outside of class with classmates.** You may pick up ideas from them, and its easier to motivate for a study session if other people are counting on you.
- **Visit the Learning Center** to study in a structured environment with a tutor.
- **Be honest with yourself.** Only you know if you really understand a concept. Only you know if you have truly been doing your best. It's easy to make an excuse to yourself as to why you can't do something. Successful people persevere despite the obstacles in their way.

One last thing before we begin. Many students who have started the semester hating math have reported starting to enjoy it after working hard enough to start understanding the material. They confess that once they began doing the math correctly with confidence, the work actually became fun! So, I say to you, "Get busy, get help, and have fun!"

HOW TO USE THIS BOOK

The book in your hands is meant to be used. At the end of the semester it will be full of your work, your notes, your sweat, and your tears. Whether your Fundamentals of Math class allows the students to use their notes on exams or not, you need the work in the book to be correct, clear to read, and organized so that you can easily refer to previous work to figure out how to do a problem

Some of the exercises or activities in the book don't leave enough space to show a clear solution.

Some of the exercises or activities in the book require that you work with physical objects, reflect on what you have learned, then summarize the ideas in clear sentences.

Some of the exercises or activities in the book ask you to fill in a table.

Often you will be asked to explain how something works rather than being told in the text how it works.

All of these require that you figure out how to show your work in such a way that both you and your instructor can clearly understand what you have done. The following suggestions may help you to find a way to do this for yourself.

- If there is very little space to show your work in the book, write your solution on a separate piece of paper.
- Any work you write on a separate piece of paper, label carefully with the page number and exercise number so that it is easy to see where it goes. Write on paper that is 3-hole punched so that you can put it in your binder next to the page it refers to.
- Do your work and write your notes on scratch paper first, then when you have checked that it is correct and checked it for clarity, re-write it as neatly as you can before putting it together in the book.

Activity 0.1 Problem Solving

Objective: To find a solution to a problem by working in a group. To share problem solving strategies with the class.

Materials: None.

Group size: 3 to 4.

Instructions: First solve the problem clearly by writing out all of your steps, then answer the questions below.

You are invited to your boss's house at 8:00 pm. At 6:30 pm you have just left the candy shop in your neighborhood where you picked up a box of candy for a hostess gift. You notice that your favorite store is having a sale. You have had your eye on a designer leather jacket for a while now. There is a line outside the store and the manager says there cannot be more than 10 people in the store at any time. You decide to wait in the line. There are 22 people ahead of you. People are leaving the store at a rate of 2 people every 5 minutes and once you get in the store, it takes you 10 minutes to find the jacket in the right size and 15 minutes to pay for it. Will you make it to your boss's house on time? Don't forget you have a 12 minute drive from the shopping center to her house.

1. What is the answer to the problem?

2. Show clearly all of the steps used to find the answer.

3. What did you do to check whether the answer is correct? Does everyone in the group agree that it is correct?

4. What pictures did you draw to help you solve the problem?

5. How did you organize the information on paper so that you could communicate your ideas clearly to your group-mates?

6. How can you write your solution clearly enough so that you will be able to follow your steps long after you've forgotten what the problem was about?

Notes:

1 Numbers

1.1 Defining Numbers

Numbers have two main uses in our world: to identify something the same way that a name does, and to represent a value. Some examples of identification numbers are account numbers, phone numbers, address numbers, social security and G-numbers, and titles or names like Chapter 5 or R2D2. Numbers that represent value are much more common, and are the subject of the study of mathematics. Some examples are anything to do with money like prices or wages, scores in sports or on an exam, measurements like area, distance, weight or speed, a time or date, and age. Often, value numbers are used to compare one object with another. For example, a jacket which costs $45 with another that costs $100; a job that offers a salary of $25,000 per year with a job that offers a salary of $40,000 per year; an apartment with an area of 500 square feet with one with an area of 800 square feet. People use value numbers to make decisions, and mathematics helps people make informed decisions.

Activity 1.1 Introducing Units: How can the meaning of the same number be different?

Objective: To learn how the meaning of numbers change based on the units assigned to them.

Materials: None.

Group size: 3 to 4.

Instructions: Use the numbers 10, 16, 0, 21, and 5 to answer the first four questions, then work on the last two.

1. Discuss together what each number in the list above reminds you of. Notice that numbers just by themselves are a very abstract concept. When numbers quantify **SOMETHING** or describe a quantity of **SOMETHING**, called a unit, they are more meaningful. Therefore Units are associated with numbers. For example: 10 *yards*, is quite different from 10 trees.
2. By yourselves, assign different units to each of the numbers 10, 16, 0, 21, and 5.
3. When everyone is done, compare your answers.
   (a) Which of the numbers did more than one person have the same units for?
   (b) Explain why that might have happened. Did those numbers HAVE to have those units to make sense, or was it just a common thing for that number to stand for?
4. Explain why it is important to include proper units with answers to word problems.
5. Tonight at home, search a newspaper for three examples of identification numbers and three examples of value numbers. For the value numbers, write a sentence explaining what object in real life is being represented with that value and include the units if there are any.
6. Tomorrow at the beginning of class, find your group members and discuss what you found.

1.2 Comparing Numbers

What determines if one number is greater than another?

We use the symbol "$>$" to indicate when one number represents more of something than another number. For instance, since 8 pounds of something is more than 5 pounds, we would write 8 lbs. $>$ 5 lbs. We use "$<$" to indicate when one number represents less of something than another number. From the same example above, we would write 5 lbs. $<$ 8 lbs.

Exercise 1.2.1 To search for the answer to this question, let's do the following exercise:

1. For each question insert either $\boxed{>}$, $\boxed{<}$, or a $\boxed{?}$ if it is unclear whether one quantity is more or less than the other.
   (a) 10 dogs ☐ 7 dogs
   (b) 10 dogs ☐ 7 cats
   (c) 10 dogs ☐ 7 buckets
   (d) \$5 ☐ 34 ¢
   (e) 2 feet ☐ 20 inches
   (f) 12 miles ☐ 3 gallons
   (g) \$42,000 per year ☐ \$5,000 per month
   (h) \$12.95 per hour ☐ \$2,500 per month
2. Which part(s) of question (1) could you clearly answer? Why?
3. What information do you need to answer the questions that were not clear? Why?
4. Explain your answers to parts (e) through (h).
5. Write a conclusion about what you need in order to compare two things using $>$ or $<$.

Activity 1.2 Powers of 10

Objective: To investigate the pattern when multiplying by powers of ten.

Materials: Calculator.

Group size: 2 to 5.

1. Use your calculator to help you find the products below.

Number of 10's	Expression	Result
2	$10 \times 10 =$	
3	$10 \times 10 \times 10 =$	
4	$10 \times 10 \times 10 \times 10 =$	
5	$10 \times 10 \times 10 \times 10 \times 10 =$	
6	$10 \times 10 \times 10 \times 10 \times 10 \times 10 =$	

2. Without using your calculator, anticipate the result of the product:

   $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 =$

3. Describe a shortcut for multiplying tens.

4. How many 10's are multiplied together to obtain the number $1,000,000,000$?

5. Find the results of the products below, then write the names of the results out in words. (Hint: For the decimals, think of money. For example, $0.1 = 0.10$ is a dime which is a tenth of a dollar, and 0.01 is a penny which is a hundredth of a dollar!)

	Expression	Result	Names in Words
(a)	$4 \times 100,000 =$	400,000	four hundred thousand
(b)	$6 \times 0.01 =$	0.06	six hundredths
(c)	$7 \times 1000 =$		
(d)	$2 \times 100 =$		
(e)	$3 \times 1000 =$		
(f)	$5 \times 10,000 =$		
(g)	$9 \times 10 =$		
(h)	$2 \times 0.1 =$		
(i)	$5 \times 0.1 =$		
(j)	$3 \times 0.01 =$		

6. What is the name of the number 407,200?

   Notice how the name of a number combines its digits with the power of ten implied by the position of the digit. The 4 represents $4 \times 100,000 = 400,000$, the 7 represents $7 \times 1000 = 7,000$ and the 2 represents $2 \times 100 = 200$.

7. What is the name of the number 53,090?
8. How much does the 5 represent in the number 53,090?
9. How much is 10 quarters worth?
10. How much is 10 dimes worth?
11. How much is 100 pennies worth?
12. How much is 1000 pennies worth?
13. How much is 1000 dimes worth?

2 Place Value

2.1 Digits

There are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Numbers are represented by placing digits together in a string. The digits in a number have different values associated with them depending on their relative position in the number.

For example:

> 276 has three digits and 7 has a value of **seven***ty* or $7 \times 10 = 70$. As you read the number, the place values are spoken as, **two** *hundred* **seven***ty*-**six**.
>
> A decimal point separates whole number values from fractional values.
> 24.89 has 4 digits and the digit 8 has the value: $8 \times 0.1 = 0.8$. For example, if it was \$24.89, the 8 would represent 8 dimes, or $8\times$\$0.1=\$0.80.

Exercise 2.1.1 Associating digits with money. \$1 bill, \$10 bill, \$100 bill, \$1000 bill

Leticia wants to buy a new motorcycle that costs \$6,327.95

1. How many digits are in 6,327.95?
2. Which is the largest digit?
3. Which digit has the largest place value?
4. What is the value of the digit with the largest value?
5. Which is the smallest digit?
6. Which digit has the smallest value and what is its value?
7. Leticia pays cash for the bike and the bills she can get from the bank are in the denominations \$10,000, \$1000, \$100, \$10, \$1, as well as dimes and pennies. How many of each denomination will she need to cover the exact price if she uses the minimum number of bills?
8. The value of each denomination of the bills Leticia used in question (7) is the place value of each digit within the number 6,327.95. Name the place value of each digit.

Note: As you see in exercise 2.1.1, there is a monetary value associated with each digit of a number according to the placement of the number. For example, the value of the digit 6 is 6 thousand since you need 6 one thousand dollar bills and the value of the 5 is 5 hundredths because you need 5 pennies and each penny is worth one hundredth of a dollar.

2.2 Introducing exponents with base 10

The following chart shows some of the place values in the number system:

one billion	$1,000,000,000 = 1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^9$
one hundred million	$100,000,000 = 1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^8$
ten million	$10,000,000 = 1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^7$
one million	$1,000,000 = 1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^6$
one hundred thousand	$100,000 = 1 \times 10 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^5$
ten thousand	$10,000 = 1 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^4$
one thousand	$1,000 = 1 \times 10 \times 10 \times 10 = 1 \times 10^3$
one hundred	$100 = 1 \times 10 \times 10 = 1 \times 10^2$
ten	$10 = 1 \times 10 = 1 \times 10^1$
one	$1 = 1 \times 1 = 1 \times 10^0$
one tenth	$0.1 = \frac{1}{10} = 1 \div 10$
one hundredth	$0.01 = \frac{1}{100} = 1 \div 100 = 1 \div 10 \div 10$
one thousandth	$0.001 = \frac{1}{1000} = 1 \div 1000 = 1 \div 10 \div 10 \div 10$

The following chart illustrates the place value of the digits:

one billion		one hundred million	ten million	one million		one hundred thousand	ten thousand	one thousand		one hundred	ten	one		one-tenth	one-hundredth	one-thousandth
	,				,				,				.			

Therefore, the number three billion, four hundred seventy five million, six hundred twenty eight thousand, five hundred eleven and nine-tenths (3,475,628,511.9) would look like this in the chart:

one billion		one hundred million	ten million	one million		one hundred thousand	ten thousand	one thousand		one hundred	ten	one		one-tenth	one-hundredth	one-thousandth
3	,	4	7	5	,	6	2	8	,	5	1	1	.	9		

Exercise 2.2.1

1. Find the place value and **value** of each digit underlined in following chart:

	Number	Place Value	Value
(a)	$23\underline{7},669$	thousands	7,000
(b)	$\underline{9},932,210$		
(c)	$1\underline{0}2$		
(d)	$86\underline{5},106$		
(e)	$65.2\underline{9}3$		
(f)	$1\underline{0}.226$		
(g)	$0.\underline{2}26$		
(h)	$\underline{6}50,900,563,115.07$		
(i)	$1127.0\underline{8}335$		

2. What number is written as forty two thousand, three hundred seventy five and twenty three hundredths?

3. What number is written as eighty billion nine hundred seven thousand four?

4. A "googol" is defined as 10^{100}, that is, 10 multiplied by itself 100 times. Describe how you could write this number on paper in standard notation.

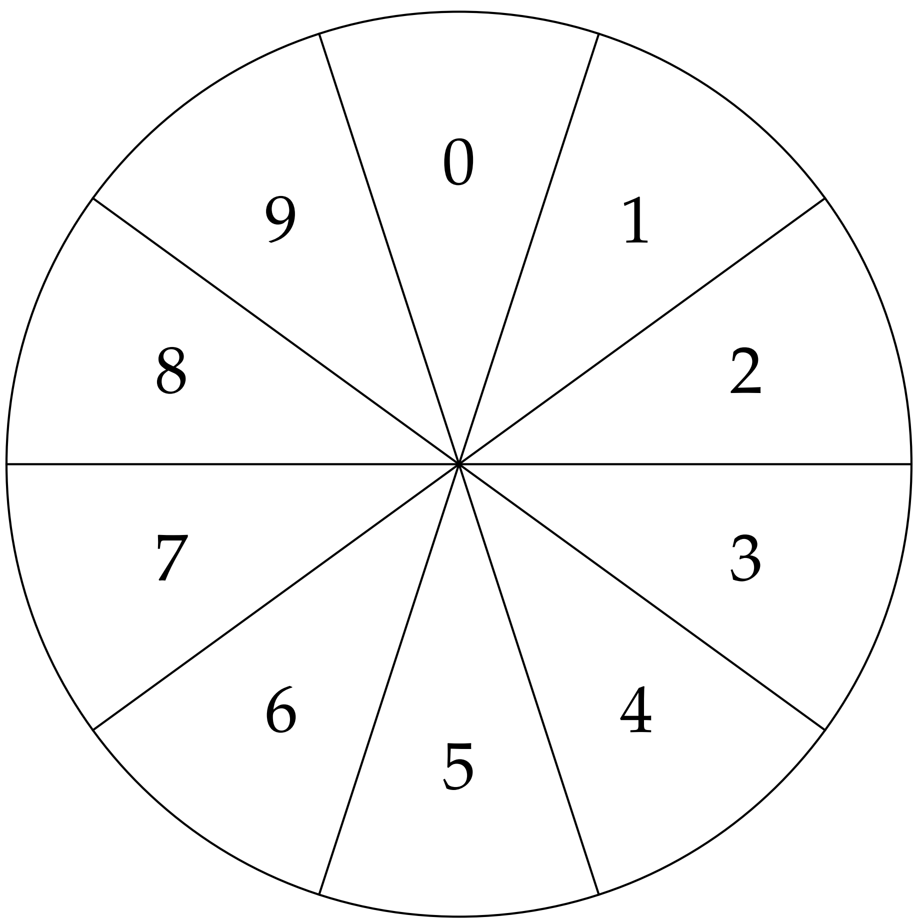
Activity 2.1 Group Game: Spin the Digits

Objective: To introduce concepts of place value by strategizing digit placement.

Materials: Paper clip, paper, pencil.

Group size: 3 to 4.

1. Each group needs a paper clip and a separate piece of paper for each person.
2. Open the paper clip from one side and use it as a spinner.
3. Each member draws 3 lines on his or her paper.
4. Each time one of the members spins the paper clip and each member records the number on one of the 3 lines that he or she has drawn.
5. After 3 spins compare your 3-digit numbers.
6. The largest 3-digit number wins the turn.



7. What is a good strategy for winning this game?

Activity 2.2 Learning Place Value with Money

Objective: To solidify concepts of place value by using money.

Materials: Bills of different denominations.

Group size: 3 to 4.

Notice that $235 could come in 235 $1 bills, or 2 $100 bills, 3 $10 bills and 5 $1 bills , or 23 $10 bills and 5 $1 bills.

1. Which combination has the fewest number of bills totaling $235? (Write how many of each denomination in the appropriate boxes.)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

2. Find three different combinations of bills/coins to make $400. (Write how many of each denomination in the appropriate boxes.)

(a)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

(b)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

(c)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

3. Find three different combinations of bills/coins to make $1. (Write how many of each denomination in the appropriate boxes.)

(a)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

(b)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

(c)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

4. Find three different combinations of bills/coins to make $7,000. (Write how many of each denomination in the appropriate boxes.)

(a)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

(b)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

(c)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

5. Find three different combinations of bills/coins to make \$5,628. (Write how many of each denomination in the appropriate boxes.)

	\$100,000	\$10,000	\$1,000		\$100	\$10	\$1		\$0.10	\$0.01
(a)				,				.		

	\$100,000	\$10,000	\$1,000		\$100	\$10	\$1		\$0.10	\$0.01
(b)				,				.		

	\$100,000	\$10,000	\$1,000		\$100	\$10	\$1		\$0.10	\$0.01
(c)				,				.		

6. Find three different combinations of bills/coins to make \$63,921.42. (Write how many of each denomination in the appropriate boxes.)

	\$100,000	\$10,000	\$1,000		\$100	\$10	\$1		\$0.10	\$0.01
(a)				,				.		

	\$100,000	\$10,000	\$1,000		\$100	\$10	\$1		\$0.10	\$0.01
(b)				,				.		

	\$100,000	\$10,000	\$1,000		\$100	\$10	\$1		\$0.10	\$0.01
(c)				,				.		

7. Find three different combinations of bills/coins to make $307,995. (Write how many of each denomination in the appropriate boxes.)

(a)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

(b)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

(c)

$100,000	$10,000	$1,000		$100	$10	$1		$0.10	$0.01
			,				.		

8. Raymond had 21 - $1000 bills, no $100 bills, no $10 bills, 26 - $1 bills, no dimes and 13 pennies. He went to the bank and exchanged them for the *fewest* number of bills/coins that represented the same amount. How much of each kind of bill/coin did he get? In the bank record below, record the value of Raymond's money and the quantity of each type of bill/coin he would have when using the fewest number of each to represent the amount.

Type of Bill/Coin	$10,000s	$1,000s	$100s	$10s	$1s	$0.1s (dimes)	$0.01s (pennies)
Count of Bill/Coin	________	________	________	________	________	________	________

How much would his money be worth?

9. What part (fraction) of $1 is a dime?

10. What part (fraction) of $1 is a penny?

11. What part (fraction) of $1 is a quarter?

2.3 Making Numbers

Exercise 2.3.1

1. When writing money in dollars and cents format, where do you put the decimal place? (between the count of which of the bills/coins?)
2. Using each digit exactly once, what would be the largest number you could make with the digits 0, 1, 2, 5, 8, 9 (with no decimal points)?
3. Why did you put the left-most digit in that spot?
4. Using each digit exactly once, what would be the smallest number you could make with the digits 0, 1, 2, 5, 8, 9 (with no decimal points)?
5. Why can't you put the smallest digit in the left-most place?
6. Using each digit exactly once, what would be the smallest 6-digit number with 2 digits after the decimal point you could make with the digits 0, 1, 2, 5, 8, 9?
7. Using each digit exactly once, what would be the largest 6-digit number with 2 digits after the decimal point you could make with the digits 0, 1, 2, 5, 8, 9?
8. If you had the digits 0, 1, 2, 5, 8, 9 to make a 6-digit number, using each digit exactly once, what would be the smallest number you could make with the 6 digits? You choose the number of digits after the decimal place.
9. What is the largest 5-digit number?
10. What is the smallest 5-digit number (without the decimal point)?
11. Using the digits 6, 4, 1, 0, and 8, exactly once each, what is the largest five digit number you can make?
12. Using the digits 6, 4, 1, 0, and 8, exactly once each, what is the largest five digit number you can make? You may include a decimal point.

2.4 Place Value and Value:

Exercise 2.4.1

1. In the number \$3,461.85, which digit has the highest place value (i.e., accounts for the most money?)
2. In the number \$2.08, which digit has the highest place value?
3. In the number \$2.08, which is the largest digit? What is its value?
4. In the number \$2.08, which digit has the highest value? What is its value?
5. In the number \$4501.32, what is the place value of the digit 0?
6. In the number \$673.47, what is the place value of the digit 4?
7. In the number \$86, 793.64, what digit is in the ten-thousands place?
8. In the number \$86, 793.64, what digit is in the hundreds place?
9. In the number \$86, 793.64, what digit is in the tenths place?
10. In the number \$86, 793.64, what is the place value of the digit 9?
11. In the number \$86, 793.64, what is the value of the digit 8?
12. In the number \$86, 793.64, what is the place value of the digit 4?
13. In the number \$86, 793.64, what is the value of the digit 4?
14. Create a number with a 9 in the thousands place.
15. Create a number with a 3 in the tens place and a 5 in the tenths place.
16. Create a number with a 2 in the hundreds place, a 1 in the hundredths place, and the digit in the ones place is twice the digit in the tens place.

2.5 Saying and Writing Numbers in Words

To read and write whole numbers: The whole numbers are the numbers in the sequence 0, 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, continuing indefinitely. When we write whole numbers in standard form we separate a group of three digits by a comma. Each group of three digits forms what is called a **period**. Each period has a particular name. We say the name of a period after we read all the numbers in that period, except the last period before the decimal point.

> **Example 1:** Read 8,412,769,215.
>
> **Solution:** Eight *billion*, four hundred twelve *million*, seven hundred sixty nine *thousand*, two hundred fifteen. (The name of each period is in *italics*. Notice that the last period doesn't have a name.)

To read and write decimal numbers: We read the whole number part exactly the same way as before, then read the decimal as "and" followed by the name of the number to the right of the decimal point using the place value of the last digit as the number's value.

> **Example 2:** Read 65,389,237,542.029.
>
> **Solution:** Sixty five billion, three hundred eighty nine million, two hundred thirty seven thousand, five hundred forty two, and twenty nine thousandths. (Notice that the name of the decimal part is *thousandths* since the last digit, the 9, is in the thousandths place.)

Exercise 2.5.1 Place commas to separate the periods, then write out the number in words:

1. 2863 ____________________
2. 76.218 ____________________
3. 28990.02 ____________________
4. 3400651899 ____________________
5. 23678.93 ____________________
6. 1000267 ____________________
7. 64822.2 ____________________

Exercise 2.5.2

1. What number is 10 more than 5,687.13?
2. What number is 400 more than 38,265.79?
3. What number is 0.3 more than 21,975.52?
4. What number is 0.05 more than 5,687.13?

Notes:

3 Rounding, Estimation, and Measurement

3.1 Rounding

Some quantities are exact, like the number of people in a room, while others are *approximations*. An approximation is a number that is close to the exact value, but has fewer non-zero digits. More zeros makes it easier to work with.

We **round** a number to a less accurate but simpler approximate value to make it easier to work with, while keeping most of the important information about the number.

For example, if we learned that the U.S. trade deficit in June 2005 was $58,753,421,989, we probably could analyze how this was important to our lives just as well if it was rounded to $59,000,000,000! In fact, without all of the non-zero digits, we could even write this number as a ”word-number” hybrid by writing, ”$59 billion”. It is usually a good idea to round a number like this, and if you listen to financial news reports, you will notice that the media often rounds numbers.

For another example, if we wanted to measure the length of a window so that we could buy the right size replacement, we could use a very accurate measuring device and find that the length is 75.18952 cm. The window store is probably satisfied with the rounded version, 75 cm. But, the window is approximately 75 cm.

Exercise 3.1.1 For the following numbers, state whether it seems reasonable that the given value is exact or an approximation. Give a reason for your answer.

1. Measurement of the distance you drive from SF to LA is 420 miles.
2. Jenisa calculated her share of the lunch that she had with her high school friends yesterday and the figure was $12.345.
3. The family’s rent each month was $1225.
4. The same family’s monthly budget for food was $840.
5. A group of friends calculated their share of the cost of a ski trip and the figure was $342.

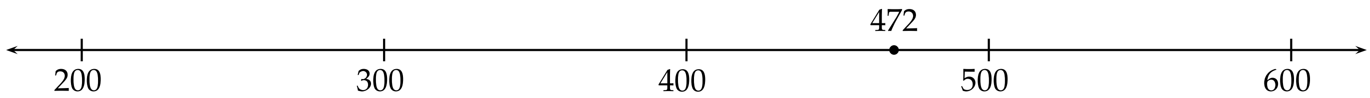
Exercise 3.1.2 For the following numbers, discuss how accurate each measurement needs to be for it to be useful, and why it requires that amount of accuracy. Also use phrases like "at least" or "at most" or "to the nearest" if appropriate for the situation.

1. Measurement for rope to tie down a load.
2. Measurement for wood in making a picture frame.
3. A cup of flour to make a cake.
4. Measurement of the length of the glass used to make the windows for a new building.
5. Scores used to compute your grade.
6. The amount of line to let out in a cast while fishing.
7. The time it will take to drive to a dinner party.
8. The amount of salt used in a pot of soup.

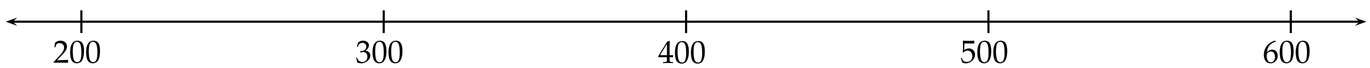
3.2 Graphing Numbers on a Number Line

Exercise 3.2.1 Graph the following numbers on the number line.

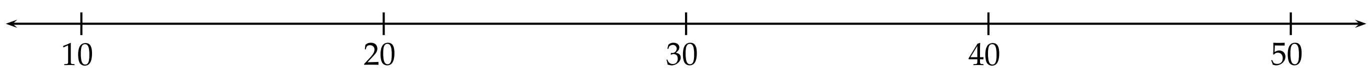
Example: To graph 472, place a dot where 472 should go, then write 472 next to the dot as in the figure:



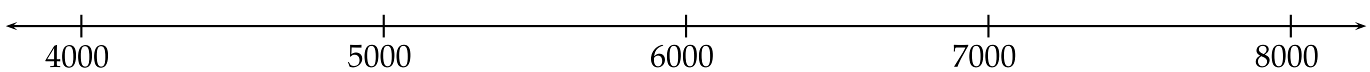
1. Graph 220, 517, 350, 490:



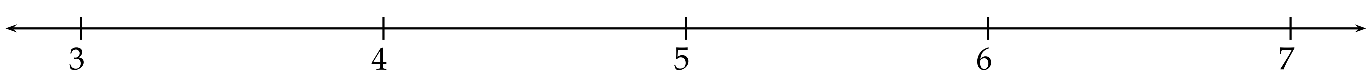
2. Graph 28, 17, 35, 46:



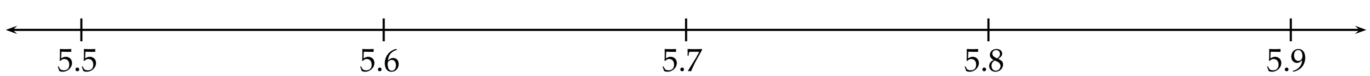
3. Graph 5593, 7200, 6500, 4207:



4. Graph 3.5, 5.2, 6.6, 4.1:

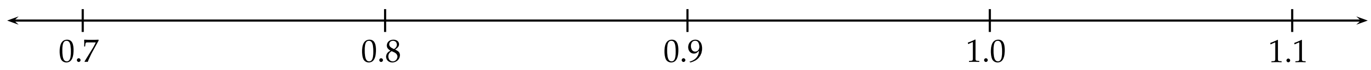


5. Graph 5.72, 5.63, 5.88, 5.55:

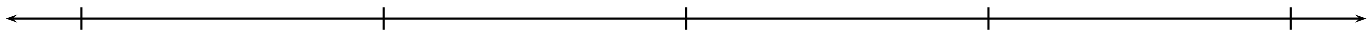


Exercise **3.2.2** Mark each number line by counting by the indicated amount.

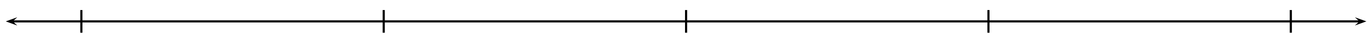
Example: To mark the number line by counting by tenths starting at 0.7, you would mark the number line like the following:



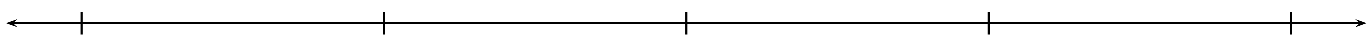
1. Count by hundreds, starting at 500:



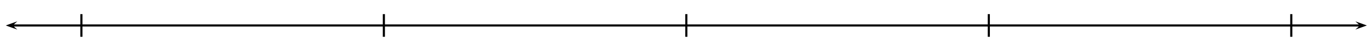
2. Count by tens, starting at 30:



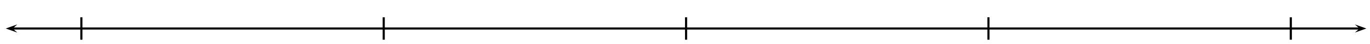
3. Count by ones, starting at 8:



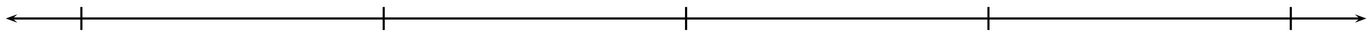
4. Count by tens, starting at 420:



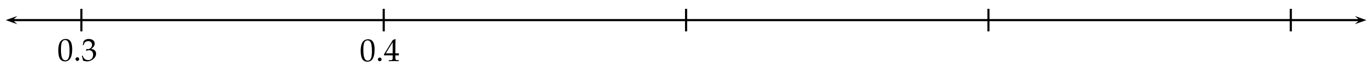
5. Count by hundreds, starting at 2100:



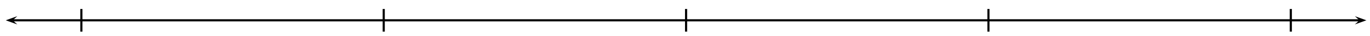
6. Count by tens, starting at 2100:



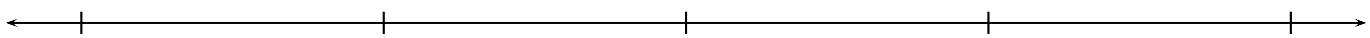
7. Count by tenths (0.1s), starting at 0.3:



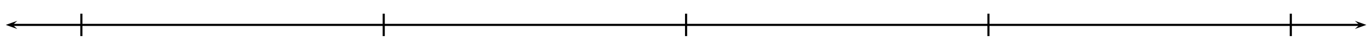
8. Count by tenths, starting at 1.8:



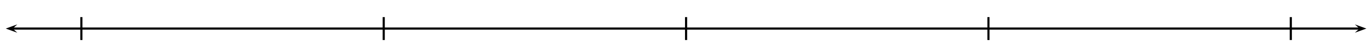
9. Count by tenths, starting at 22.5:



10. Count by hundreds, starting at 34,800:



11. Count by thousands, starting at 998,000:



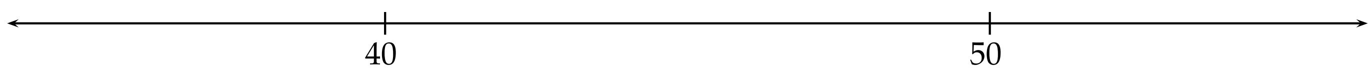
3.3 Rounding to a Particular Decimal Place:

To round numbers to a particular decimal place, do the following:

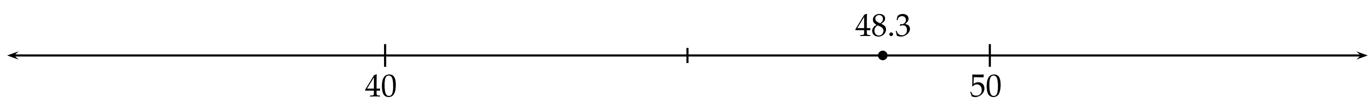
1. Determine the correct decimal place (tenths, ones, tens, hundreds, etc.)
2. Draw a number line, and count by the determined decimal place starting at the number just below the number you are rounding, and ending with the number just after it.
3. Graph the number that you are rounding on the number line.
4. The rounded value is the count number that is closest to your graphed number. If the graphed number is exactly in the middle of two count numbers, round to the higher number.

Example: Round 48.3 to the nearest ten.

Solution: To round 48.3 to the nearest ten, count by tens starting at 30 like the following:



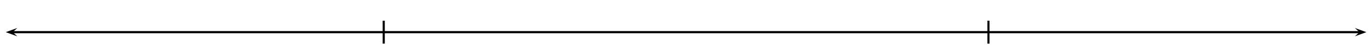
Then, graph 48.3, and determine which number is closest...it helps to put a tick mark at the half-way points:



48.3 is to the right of the half-way point between 40 and 50, so it's closer to 50. Therefore, 48.3 rounded to the nearest ten is 50.

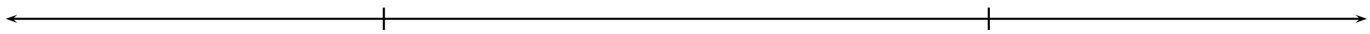
Exercise 3.3.1 Round the following as indicated.

1. Round 286 to the nearest hundred:



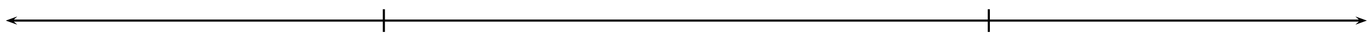
Final answer:_______________

2. Round 5.37 to the nearest one:



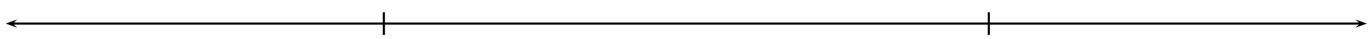
Final answer:________________

3. Round 5.37 to the nearest tenth:



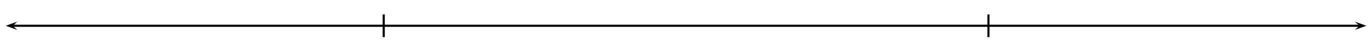
Final answer:________________

4. Round 5.37 to the nearest ten:



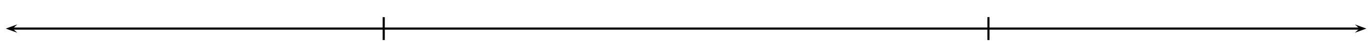
Final answer:________________

5. Round 5.37 to the nearest hundred:



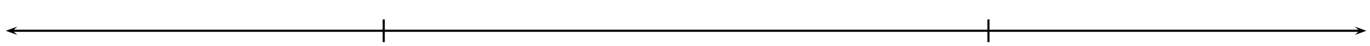
Final answer:________________

6. Round 2358 to the nearest hundred:



Final answer:________________

7. Round 2358 to the nearest thousand:



Final answer:________________

Exercise **3.3.2** Fill in the table to practice rounding. The first two are done for you as examples:

The Number to Round off and the Level of Rounding	**Underline the Digit(s)**	**The Next Digit**	**Round Up or Down**	**Answer**
1) (example) 18.09 Round to the nearest 10	$\underline{1}8.09$	8	Up, since $8 \geq 5$	20
2) (example) 23.032 Round to the nearest 100th	$23.0\underline{3}2$	2	Down, since $2 < 5$	23.03
3) 5671.98 Round to the nearest 100				
4) 5671.983 Round to the nearest 100th				
5) 1098.67 Round to the nearest 100				
6) 36.0409 Round to the nearest 100th				
7) 99.9986 Round to the nearest 1				
8) 99.9986 Round to the nearest 10th				
9) 0.00098 Round to the nearest 100th				
10) 6.85 Round to the nearest 10th				

Exercise **3.3.3** For each table of data, make a new table by rounding values as indicated:

1. The following table lists the number of pages in each of the books in the Harry Potter series. Fill in the new table with the number of pages rounded to the nearest 10.

Given Data Table

Book Number	**Number of Pages**
1	309
2	341
3	435
4	734
5	870
6	652

Rounded Data Table

Book Number	**Number of Pages**
1	
2	
3	
4	
5	
6	

2. The following table lists the numbers of new products containing the artificial sweetener Splenda for various years. Fill in the new table with the number of new products containing Splenda rounded to the nearest 100.

Given Data Table

Year	Number of New Products Containing Splenda
2000	183
2001	261
2002	365
2003	561
2004	1330

Rounded Data Table

Year	Number of New Products Containing Splenda
2000	
2001	
2002	
2003	
2004	

3. The following table lists the percentages of Americans who have earned a college degree for various years. Fill in the new table with the percentages of Americans who have earned a college degree rounded to the nearest whole percentage.

Given Data Table

Year	Percentage of Americans Earning a College Degree
1960	7.7
1970	10.7
1980	16.2
1990	21.3
2000	25.6
2003	27.2

Rounded Data Table

Year	Percentage of Americans Earning a College Degree
1960	
1970	
1980	
1990	
2000	
2003	

4. The more you stretch a spring with your hands, the more *force* the spring exerts on your hands. The following table compares the amount of stretch with the forces exerted. Fill in the new table with the amount of stretch data rounded to the nearest 100th.

Given Data Table

Force (newtons)	Stretch (meters)
0.5	0.018
1.0	0.035
1.5	0.052
2.0	0.069
2.5	0.087
3.0	0.104
3.5	0.121
4.0	0.139
4.5	0.156
5.0	0.173

Rounded Data Table

Force (newtons)	Stretch (meters)
0.5	
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	

3.4 Estimation and Measurement

Exact or Estimate

When you are finding out the value of a quantity in real life (as opposed to reading a given value in a book), some values you know to be **exact**, while others you must **estimate** to obtain an **approximation**. Some examples of exact values are as follows:

- The number of people at a small dinner party.
- The number of children in a family.
- The number of hours in a day.
- The price of gas at a gas station.
- The number of grams in a kilogram.
- The number of units passed in a semester.

By contrast, the following is a list of values that must be estimated:

- The time it takes to complete an assignment.
- The number of people who live in San Francisco.
- The weight of a sick dog at the vet.
- The distance from SF to LA.
- A person's height.

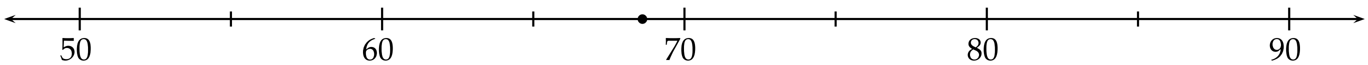
Exercise 3.4.1

1. Explain what makes the values in the first list exact values, and the values in the second list approximate values.
2. State five things that have a value that is exact.
3. State five things that have a value that must be estimated.
4. For the following quantities, give a value for the quantity, then state whether it is exact or an estimation:
   (a) The number of people in class on the most recent day.
   (b) The height of the classroom door.
   (c) The length of your index finger.
   (d) The number of typed words on this page.
   (e) The weight (in pounds) of the dog you have most recently seen.

Estimating Points on a Number Line

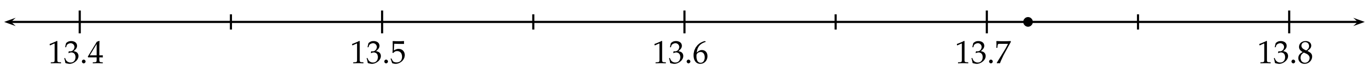
Exercise 3.4.2 For the following, use the number lines to estimate the value of the marked point.

Example 1:



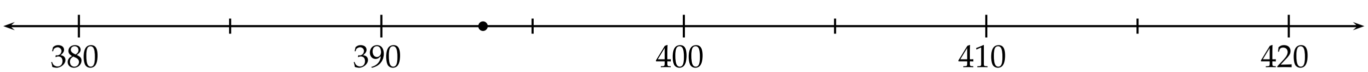
Solution: The value of the point is approximately 68. Since the value of the marked numbers are counting by tens, we only have confidence in one more decimal place. Therefore, we estimate to the nearest whole number.

Example 2:

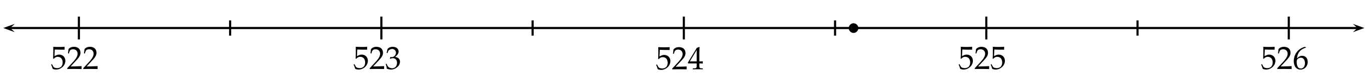


Solution: The value of the point is approximately 13.72. Since the value of the marked numbers are counting by tenths, we only have confidence in one more decimal place. Therefore, we estimate to the nearest hundredth.

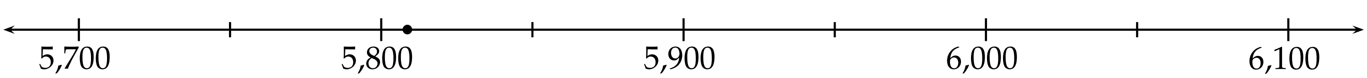
1.



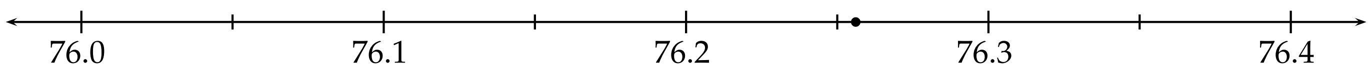
2.



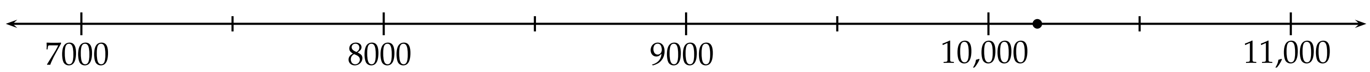
3.



4.



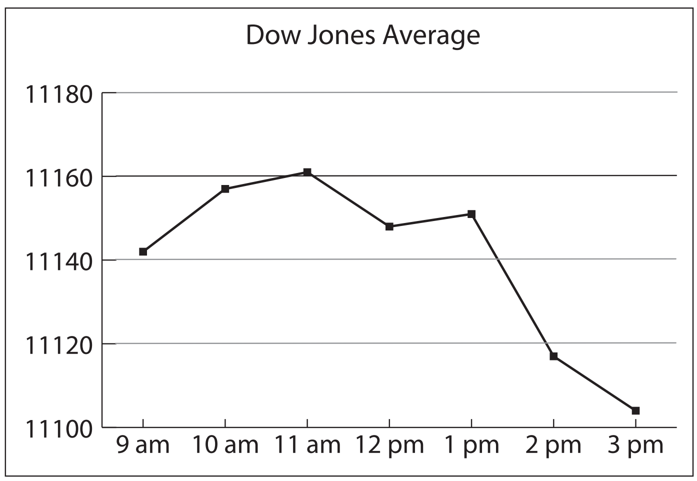
5.



Estimating Values from Line and Bar Graphs

Exercise 3.4.3 Estimate the values from the graphs.

1. Use the following graph to estimate the value of the Dow Jones Average at certain times.



Example: Estimate the value of the Dow at 10 am.

Solution: The value of the Dow at 10 am is estimated to be 11,158.

(a) Estimate the value of the Dow at 9 am.

(b) Estimate the value of the Dow at 11 am.

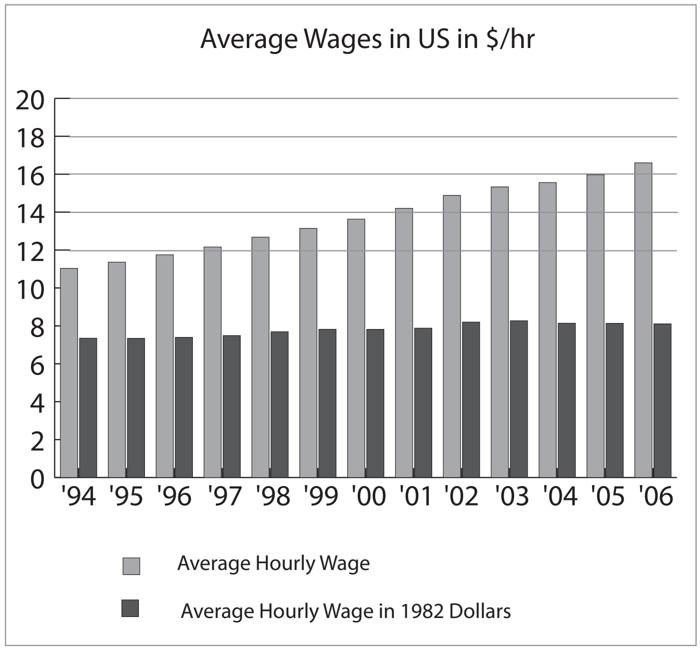
(c) Estimate the value of the Dow at 12 pm.

(d) Estimate the value of the Dow at 1 pm.

(e) Estimate the value of the Dow at 2 pm.

(f) Estimate the value of the Dow at 3 pm.

2. Use the following graph to estimate the average U.S. wage (in dollars per hour) in June of different years.



Example: Estimate the average U.S. wage in June 1997.

Solution: The average U.S. wage in June 1997 is estimated to be $12.10 per hour.

(a) Estimate the average U.S. wage in June 1994.

(b) Estimate the average U.S. wage in June 1996.

(c) Estimate the average U.S. wage in June 1999.

(d) Estimate the average U.S. wage in June 2001.

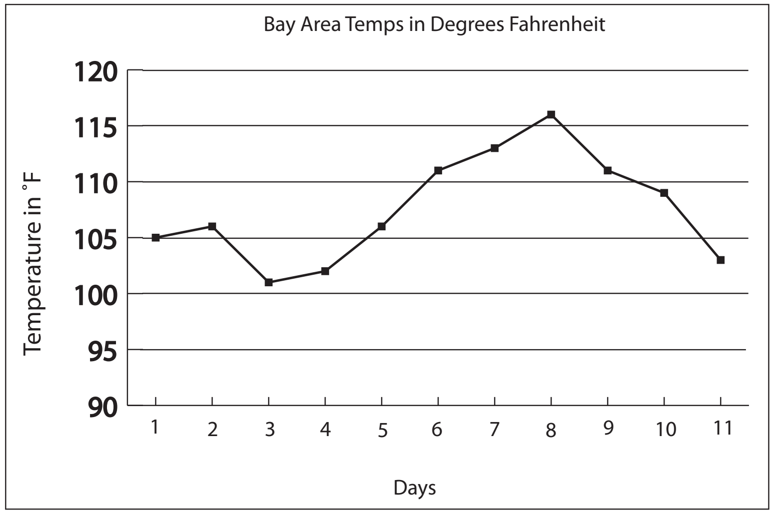
(e) Estimate the average U.S. wage in June 2004.

(f) Estimate the average U.S. wage in June 2006.

(g) When were wages about $15 per hour?

(h) What do the "Ave. Wage in 1982 Dollars" bars tell you?

3. Use the following graph to estimate the high temperatures during a record breaking eleven day heat wave in the Bay Area.



(a) Estimate the high temperature on day 1.

(b) Estimate the high temperature on day 3.

(c) Estimate the high temperature on day 6.

(d) Estimate the high temperature on day 7.

(e) Estimate the high temperature on day 10.

(f) On what day was the highest Bay Area high temperature recorded?

(g) On what day was the lowest Bay Area high temperature recorded?

(h) Between which days is the temperature going up?

An Introduction to Measurement

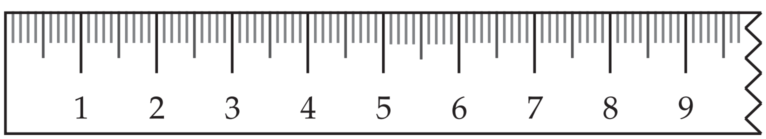
When we are estimating the length of something, we can get a more accurate measurement by using a tool. One might use a ruler to measure the length of a finger, a tape measure to measure the length of a window sill, and an odometer to measure the distance from SF to LA. In the U.S., the units of length are the inch, the foot, the yard, and the mile. We measure with units that are appropriate for the lengths. For example, we measure the distance between two cities in miles, the length of a book in inches, and the length of a room in feet. In this section, you will be using the **metric** system to measure length, and the most appropriate metric system unit to use for the following lengths is the **centimeter**.

We could use an inch ruler, but inches are broken down into fractions of an inch rather than tenths. We will wait to introduce inches until after we have introduced fractions.

To use a ruler to measure length, line the end of the ruler at one end of the line, and note how far along the ruler the other end of the line is.

Example 1: Measure the length of the following line accurate to one-tenth of a centimeter:

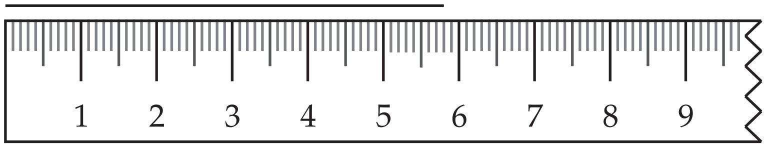
Solution: We place the ruler near to the line as follows:



Looking at where the line ends, we estimate the length as 4.6 cm. The 4 stands for 4 whole centimeters which is indicated by the 4 on the ruler. The .6 (6 tenths) stands for 6 out of ten parts of the next centimeter. We know this because the end of the line is above the 6th mark past the 4.

Example 2: This time, measure the length of the following line accurate to one whole centimeter:

Solution: We place the ruler near to the line as follows:



Since the end of the line is closer to 6 cm than it is to 5 cm, we say the line measures approximately 6 cm accurate to the nearest whole centimeter.

Exercise 3.4.4 Using a centimeter ruler, measure the lengths of the following lines accurate to the nearest whole centimeter. Include appropriate units in your answers.

1.

2.

3.

4.

5.

Exercise 3.4.5 Using a centimeter ruler, measure the lengths of the following lines accurate to the nearest one-tenth of a centimeter. Include appropriate units in your answers.

1.

2.

3.

4.

5.