1. Determine the following antiderivatives.

a) $\displaystyle\int \frac{x^2+1}{\sqrt{x}}\,\mathrm{d}x$ b) $\displaystyle\int \frac{x^2}{x^3+1}\,\mathrm{d}x$ c) $\displaystyle\int \frac{x^2}{x-4}\,\mathrm{d}x$

2. The table below gives the surface area of water at various depths in a bathtub. Use a trapezoid approximation with $n = 6$ intervals to help you approximate the volume of water in the tub.

depth (h) in feet	0	.5	1	1.5	2	2.5	3
SA ($A(h)$) in ft^2	0	7.4	10.8	11.1	13.0	13.6	13.8

3. For which functions does the trapezoid approximation give exact answers?

4. Suppose midpoint approximations to a definite integral yield the following results: $M(30) = 4.039, M(10) = 3.955$. What is the exact value of the integral?

5. Determine whether the following integrals diverge or converge. If they converge, find the value to which they converge.

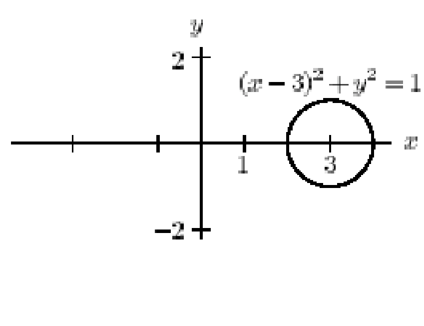
a) $\displaystyle\int_0^1 \frac{1}{x^{\frac{417}{418}}}\,\mathrm{d}x$ b) $\displaystyle\int_0^\infty 3x^2e^{-x^3}\,\mathrm{d}x$ c) $\displaystyle\int_0^2 \frac{1}{\sqrt{4-x^2}}\,\mathrm{d}x$

6. Do these integrals converge or diverge? Explain.

a) $\displaystyle\int_0^\infty \frac{e^{-x}}{1+x^2}\,\mathrm{d}x$ b) $\displaystyle\int_0^\infty \frac{e^{-x}}{1+x}\,\mathrm{d}x$

7. Determine the volume of the solid formed by revolving the region in the first quadrant bounded by $y = x^3$ and $y = x$ about the axis $y = 2$.

8. Find the volume of the torus (doughnut) formed by rotating $(x-3)^2 + y^2 = 1$ about the y–axis.



9. Determine the length of the arc between $x = 1$ and $x = 3$ for $f(x) = \sqrt{4-x}$.

10. When an oil well burns, sediment is carried up into the air by the flames and is eventually deposited on the ground. Less sediment is deposited further away from the oil well. Experimental evidence indicates that the density (in tons/square mile) at a distance r from the burning oil well is given by

$$\delta(r) = \frac{7}{1+r^3}$$

Find and evaluate an integral which represents this total deposit.

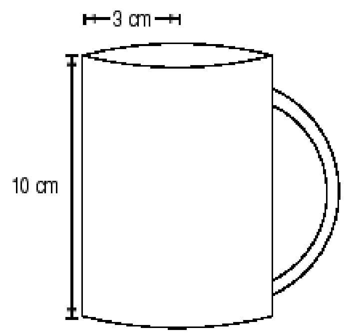
11. The globular cluster M13 is a spherical distribution of stars which orbits our galaxy. Suppose that the density of stars in the cluster is purely a function of distance, r, from the center of the cluster and is given as

$$\rho(r) = \left(1 + \left(\frac{r}{100}\right)^3\right)^{-5}$$

where r is measured in light-years, and $0 \le r \le 100$ ly. (One light-year is the distance light travels in one year; lightyear is abbreviated as ly.)

Set up an integral whose value is the exact number of stars in M13.

12. A cylindrically-shaped mug with a 3 cm radius and a 10cm height is filled with tea. You have added some sugar to the tea, which tends to settle to the bottom of the mug. It turns out that the density ρ of sugar (in gm/cm^3) in the tea, as a function of the height, h, in cm, above the bottom of the mug, is given by the formula $\rho(h) = 0.01(10 - h)$.



Write and evaluate the integral that gives the exact amount of sugar in the mug.

13. You have a bank account that earns 8% nominal annual interest compounded continuously, and you want to have \$80,000 in the bank account in five years so that you can buy a brand new Porsche.

(a) How much money would you have to deposit in one lump sum today so that the account balance would be \$80,000 in five years?

(b) If you instead deposit money in the account at a constant continuous rate of K dollars per year, then write an integral in terms of K that approximates the balance of the account after five years. Evaluate the integral in terms of the constant K.

(c) At what constant continuous rate Kdollars per year would you have to deposit money so that the balance of the account would be \$80,000 after five years?

14. The weight density (lb.s/ft^3) of a sphere with radius 10ft is inversely proportional to the distance from the bottom of the sphere according to the the relation: $\rho(y) = \frac{1}{y+1}$. Find the weight of the sphere.

17. Find the exact value of $\frac{3}{7} + \left(\frac{3}{7}\right)^2 + \cdots + \left(\frac{3}{7}\right)^{100}$

18. Does the series $4 + \frac{4}{\sqrt{3}} + \frac{4}{3} + \frac{4}{3^{3/2}} + \frac{4}{3^2} + \cdots$ converge or diverge?

19. Determine whether the series below converges or diverges.

$$\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$$

20. Determine whether the series below converges or diverges.

$$\sum_{n=0}^{\infty} \frac{1}{4n^4 + e^n}$$

21. Find the radius of convergence for the series

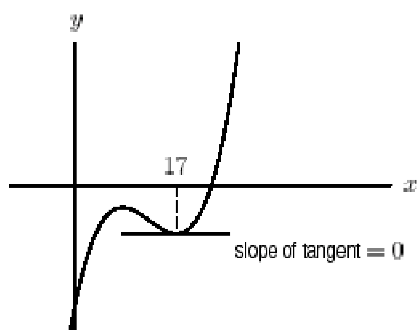
$$\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n+1}}$$

22. Find the interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$$

23. Construct the Taylor polynomial approximation of degree 3 to the function $f(x) = \arctan x$ about the point $x = 0$. Use it to approximate the value $f(0.25)$. How does the approximation compare to the actual value?

24. The graph of $y = f(x)$ is given below.



Suppose we approximate $f(x)$ near $x = 17$ by the second degree Taylor polynomial

$$a + b(x-17) + c(x-17)^2$$

Determine the sign of each coefficient ($a, b,$ and c) and explain your reasoning.

25. Write the fifth degree Taylor Polynomial approximating

$$\int \cos(t^3)\, dt$$

26. What is the exact value of the series $3 - \frac{3^2}{2} + \frac{3^3}{3} - \frac{3^4}{4} + \cdots$?

27. According to the theory of relativity, the energy, E, of a body of mass m is given as a function of its speed, v, by

$$E = mc^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

where c is the (constant) speed of light.

(a) Assuming $v < c$, expand E as a series in $\frac{v}{c}$, as far as the second nonzero term.

(b) Explain why the series shows you that if $\frac{v}{c}$ is very small, E can be well approximated by $E \approx \frac{1}{2}mv^2$.

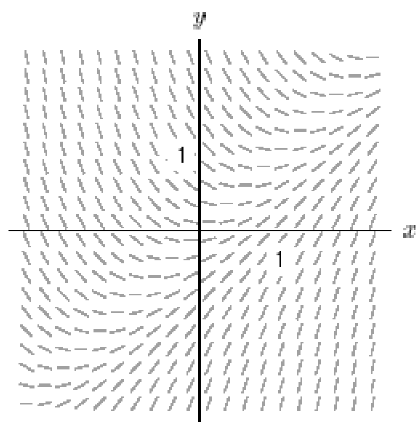
28. Give a bound for the maximum possible error for the nth degree Taylor polynomial about $x = 0$ approximating $\sin \frac{x}{2}$ on the interval $[0, 1]$.

29. Find the interval of convergence for the Taylor Series given by

$$f(x) = 2(x-3) - \frac{2^2(x-3)^2}{2} + \frac{2^3(x-3)^3}{3} - \frac{2^4(x-3)^4}{4} + \cdots$$

30. Show that $y = 3\cos(3t)$ satisfies the differential equation $y'' + 9y = 0$.

31. The slope field for the differential equation $\frac{dy}{dx} = x - y$ is shown below.



On the slope field, sketch the solution curve to the differential equation starting at $x = 0, y = 1$ and ending at $x = 1$. From your sketch, approximate the value of y when $x = 1$.

32. Consider the differential equation

$$\frac{dy}{dx} = x^2 + y$$

Beginning at the point $(1, 3)$ use Euler's method with $\Delta x = 0.5$ to approximate the value of $y(2)$. Show all your steps and explain whether this is an overestimate or and underestimate of the true value.

33. Separate variables to help you solve the differential equations below (subject to the initial condition $y(0) = 1$).

$$\text{a)}\frac{dy}{dx} = \frac{4}{1+x^2} \qquad \text{b)}\frac{dy}{dx} = \sqrt{xy} \qquad \text{c)}\frac{dy}{dx} = \frac{\cos^2 y}{x}$$

34. Consider the Hakosalo residence in Oulu, Finland. Assume that heat is lost from the house only through windows and the rate of change of temperature in $°F/h$ is proportional to the difference in temperature between the outside and the inside. The constant of proportionality is $\frac{1}{29}$. Assume that it is 10°F outside constantly. On a Thursday at noon the temperature inside the house was 65°F and the heat was turned off until 5 pm.

(a) Write a differential equation which reflects the rate of change of the temperature in the house between noon and 5 pm.

(b) Find the temperature in the house at 5 pm. (You may do this analytically or using your calculator to get a rough estimate.)

35. The population of aphids on a rose plant increases at a rate proportional to the number present. In 3 days the population grew from 800 to 1400.

(a) Write down a differential equation for the population of aphids at time t in days, where t = 0 is the day when there were 800 aphids.

(b) How long does it take for the population to get 10 times as large?

(c) What was the population on the day before there were 800?

36. There is a theory that says the rate at which information spreads by word of mouth is proportional to the product of the number of people who have heard the information and the number who have not. Suppose the total population is N.

(a) If $p = f(t)$ is the number of people who have the information, how many people do not have the information?

(b) Write and solve a differential equation that describes the rate, $\frac{dp}{dt}$, at which the information spreads by word of mouth.

(c) Why does this theory make sense?

(d) Sketch the graph of $p = f(t)$ as a function of time.

37. Use a Taylor polynomial of degree 5 to find an approximate solution to the differential equation below, subject to the given initial conditions.

$$y'' - xy' = 0 \qquad\qquad y(0) = 1,\ y'(0) = 1$$