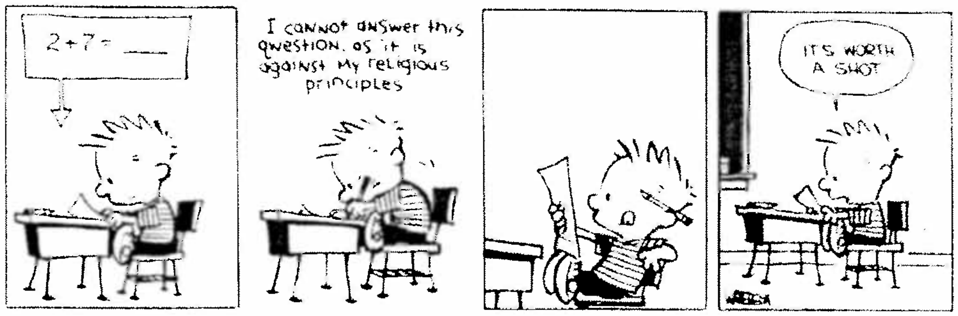
Methods Exam 1 Name:____________________

Show all relevant work!

YOU MAY NOT USE A CALCULATOR ON THIS EXAMINATION



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1. Find $\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,\mathrm{d}x$.

let $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{dx}{\sqrt{x}}$

$= 2\int 2^{u}\,du = \frac{2}{\ln 2}\,2^{\sqrt{x}} + C$

2. Show $\int \sin^2\theta\,\mathrm{d}\theta = \frac{1}{2}(\theta - \sin\theta\cos\theta) + C$.

$u = \sin\theta \quad v' = \sin\theta$

$u' = \cos\theta \quad v = -\cos\theta$

$\int \sin^2\theta\,d\theta = -\sin\theta\cos\theta + \int \cos^2\theta$

$\int \sin^2\theta\,d\theta = -\sin\theta\cos\theta + \int 1 - \sin^2\theta \cdot d\theta$

$\int \sin^2\theta\,d\theta = -\sin\theta\cos\theta + \int d\theta - \int \sin^2\theta\,d\theta$

$2\int \sin^2\theta\,d\theta = -\sin\theta\cos\theta + \theta$

$\int \sin^2\theta\,d\theta = \frac{1}{2}\left[\theta - \sin\theta\cos\theta\right] + C$

3. Find $\int x^2\sqrt{x-2}\,\mathrm{d}x$.

SUB

$u = x-2 \longrightarrow u+2 = x$

$du = dx$

$= \int (u+2)^2 u^{\frac{1}{2}}\, du$

$= \int (u^2+4u+4)u^{\frac{1}{2}}\, du$

$= \int u^{5/2} + 4u^{3/2} + 4u^{\frac{1}{2}}\, du$

$= \frac{2}{7} u^{7/2} + \frac{8}{5} u^{5/2} + \frac{8}{3} u^{3/2} + C$

$= \frac{2}{7}(x-2)^{7/2} + \frac{8}{5}(x-2)^{5/2} + \frac{8}{3}(x-2)^{3/2} + C$

PARTS

u		v'
x^2	$+$	$(x-2)^{1/2}$
$2x$	$-$	$\frac{2}{3}(x-2)^{3/2}$
2	$+$	$\frac{4}{15}(x-2)^{5/2}$
0		$\frac{8}{105}(x-2)^{7/2}$

$= \frac{2}{3}x^2(x-2)^{3/2} - \frac{8}{15}x(x-2)^{5/2} + \frac{16}{105}(x-2)^{7/2} + C$

4. Find $\int \frac{z+1}{z-1}\,\mathrm{d}z$.

$$\int \frac{z+1}{z-1}\,dz = \int \frac{z-1+2}{z-1}\,dz = \int \frac{z-1}{z-1} + \frac{2}{z-1}\,dz$$

$$= \int 1 + \frac{2}{z-1}\,dz$$

$$= \boxed{z + 2\ln|z-1| + C}$$

x	2	3	4	5
$f(x)$	5	7	2	-1
$f'(x)$	3	4	5	2
$f''(x)$	-1	2	4	-3

5. Use the table above to find $\int_2^5 x f''(x)\,dx$

$u = x \qquad dv = f''$

$du = dx \qquad v = f'$

$\int_2^5 x f'' \longrightarrow x f' \Big|_2^5 - \int_2^5 f'\,dx$

$= x f'(x) - f(x) \Big|_2^5$

$= [5(2) - (-1)] - [2(3) - 5]$

$11 - [1]$

$= \boxed{10}$

6. Show that $\int \frac{1}{P - P^2}\,dP = \ln\left(\frac{|P|}{|P-1|}\right) + C$

$\frac{1}{P(1-P)} = \frac{A}{P} + \frac{B}{1-P}$

$A(1-P) + BP = 1$

$A = 1$

$B - A = 0 \longrightarrow B = 1$

$\int \frac{1}{P} + \frac{1}{1-P}\,dP$

$= \int \frac{1}{P} - \frac{1}{P-1}\,dP$

$= \ln|P| - \ln|P-1| + C$

$= \ln \frac{|P|}{|P-1|} + C$

7. Find $\int \frac{1}{\sqrt{9-x^2}}\,dx$

let $x = 3\sin\theta \longrightarrow x^2 = 9\sin^2\theta$

$dx = 3\cos\theta\, d\theta$

$$\int \frac{3\cos\theta\, d\theta}{\sqrt{9-9\sin^2\theta}} = \frac{3}{3}\int \frac{\cos\theta}{\sqrt{1-\sin^2\theta}}\,d\theta$$

$$= \int 1\, d\theta$$

$$= \theta$$

$$= \boxed{\arcsin(x/3) + C}$$

ALT:

$$\int \frac{1}{\sqrt{9-x^2}}\,dx = \int \frac{1}{\sqrt{9(1-\frac{x^2}{9})}}\,dx$$

$$= \frac{1}{3}\int \frac{1}{\sqrt{1-(\frac{x}{3})^2}}\,dx$$

let $u = x/3 \rightarrow du = \frac{dx}{3}$

$3\,du = dx$

$$\frac{3}{3}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= \arcsin u + C$$

$$= \arcsin(\tfrac{x}{3}) + C.$$