Notes for Integration

0. (a) $\displaystyle\int \frac{1}{x^2+1}\,\mathrm{d}x = \arctan x + C$; $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x = \arcsin x + C$

(b) $\displaystyle\int \frac{x}{x^2+1}\,\mathrm{d}x$ let $u = x^2+1$,

$\displaystyle\int \frac{x^2}{x^2+1}\,\mathrm{d}x$, try re-writing as $\displaystyle\int \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\,\mathrm{d}x = \int 1\,\mathrm{d}x - \int \frac{1}{x^2+1}\,\mathrm{d}x$

$\displaystyle\int \frac{1}{x^2-1}\,\mathrm{d}x$ Use partial fractions.

1. Identify an effective method of integration. Careful, some of these have no antiderivative formula.

(a) $\int x(\ln x)^2\,\mathrm{d}x$ By parts – twice.
(b) $\int xe^{x^2}\,\mathrm{d}x$ Substitution.
(c) $\int x^2e^{x^2}\,\mathrm{d}x$ No antiderivative formula.

(d) $\int x\sin x\,\mathrm{d}x$ By parts.
(e) $\int \frac{x}{\ln x}\,\mathrm{d}x$ No antiderivative formula.
(f) $\int \frac{e^x}{e^x+1}\,\mathrm{d}x$ Substitution.

(g) $\int \frac{e^x+1}{e^x}\,\mathrm{d}x = \int 1 + e^{-x}\,\mathrm{d}x$ etc.
(h) $\int \frac{2x+1}{\sqrt{1-3x}}\,\mathrm{d}x$ Substitution (or parts).

2. Find the area between the graphs of $y = \ln x$ and $y = x - 2$. You may use your calculator but be sure to show how you set up the problem.

Ans: ~ 1.95

3. The function $V = V_0\cos(120\pi t)$ gives a reasonable model of the voltage produced by alternating current generators.

(a) Find the average (mean) value of V over one full cycle. Comment on the result.

Ans: 0. Should make sense since function has equal areas above and below t–axis over 1 period.

(b) Find (using the FTC) the Root–Mean–Square (RMS) value for one full cycle of V: $\sqrt{\frac{1}{1/60}\int_0^{1/60} V_0^2\cos^2(120\pi t)\,\mathrm{d}t}$

Ans: $\dfrac{V_0}{\sqrt{2}}$

(c) What is the significance of V_0 in this context? **Ans:** Amplitude. The maximum voltage.

(d) Determine (to one decimal place) the value of V_0 that will produce an RMS voltage of 120 VAC.

Ans: 169.7 VAC