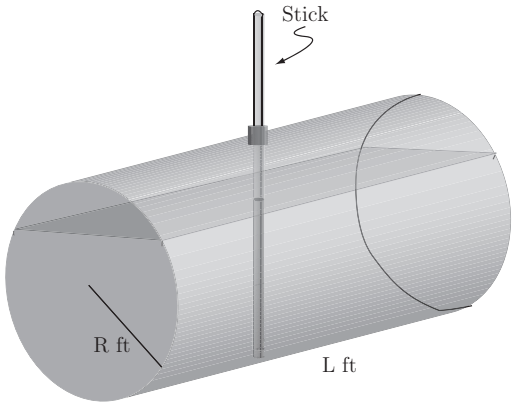
Calculus 2 Spring Project

You have just been hired by the Environmental Protection Agency (EPA) under the Superfund Program to measure the level of toxic wastes in buried tanks across New Mexico. Most of these tanks are cylinders, with their axes horizontal. You are to "stick the tank" by inserting a measuring rod through a hole in the center of the top until it touches the bottom, then pulling it out and reading off the liquid level showing on the stick. They have hired you because you know calculus; they have faith that you can convert the "height on the stick" to the "filled volume of the tank."



Problems to solve:
Assuming that the cross-sectional radius of the tank is R and its length is L, calibrate the stick for them. That is, provide a formula to convert height showing on the stick to volume of liquid. Check your results by doing the calculation in two separate ways:

(a) Evaluate a definite integral that gives the filled volume in terms of the height, h, on the stick. (You may receive additional points for evaluating your integral the long way, by using the trigonometric substitution $h = \sin\theta$ and appropriate diagrams).

(b) Use elementary geometry and trigonometry (no calculus) to obtain the volume.

(c) Show that your results for (a) and (b) are the same. (if you choose the right representation for (a) this will be trivial.

Bonus: Write field directions for calibrating a blank stick with appropriate markings assuming the Radius and Length of the tank to be measured is known. Markings should give related volumes in terms of common fractions ($\frac{1}{5}$'s, $\frac{1}{4}$'s , and $\frac{1}{3}$'s) of the diameter of the tank.

Completed Project:
When you have done the work necessary to complete the project, you need to prepare it in written form. The paper you turn in should have a mix of equations formulas and prose to support your conclusions. Use complete sentences, good grammar and correct punctuation. The prose should be written in order to convey to the reader an explanation of what you have done. It should be written in such a way that it can be read and understood by anyone who knows the material in this course. You will be graded on your written presentation as well as the mathematical content.

Adapted from Cohen, Gaughan, et al. *Student Research Projects in Calculus*; MAA: Washington, D.C., 1991