

$$\int_1^x \frac{1}{t} dt \text{ Should Equal } \ln x^1$$

Introduction:

When we consider the integrals of t^r , all antiderivatives are of the same form with the exception of the case where $r = -1$. This is often confusing, even though the result for $r = -1$ is consistent with all the other results. In this problem you are going to demonstrate that this is the case. You will need to be familiar with properties of logarithms and L'hospital's Rule.

The Problem:

- Let $F(r) = \int_1^2 t^r dt$, with r a real number not equal to -1 . Evaluate $F(r)$ for several values of r close to -1 . (You will need your calculator for this). Do the values of $F(r)$ seem to approach a limit? (Choose enough values near -1 to allow you to see a pattern.) Do you recognize this limiting value? Replace the upper bound of the integral, 2, with a 3, then a 4, and finally a 10, then answer the same questions for each.
- Let b be a fixed positive number. For r a real number not equal to -1 , redefine the function F by $F(r) = \int_1^b t^r dt$. Find a simpler form of $F(r)$.
- Show that F is a continuous function. Give a good justification for your answer.
- How should we define $F(-1)$ so that F is continuous at -1 ? Show that this value makes F continuous at -1 . Reread the introductory paragraph and explain the title of this project.
- Explain your results from part (a) in light of your discoveries in part (d).

Completed Project:

When you have done the work necessary to complete the project, you need to prepare it in written form. The paper you submit should have a mix of equations, formulas, diagrams, and prose to support your conclusions. Use complete sentences, good grammar, and correct punctuation. The prose should be written in order to convey to the reader an explanation of what you have done. It should be written in such a way that it can be read and understood by anyone familiar with the material in this course. You will be graded on your written presentation as well as the mathematical content. Be sure to include all of your reasoning and cite an resources you used in finding your solution.

¹Reproduced from Cohen, Gaughan, et al. *Student Research Projects in Calculus*;MAA: Washington, D.C.,1991