

Show all relevant work!

1. Evaluate or show divergent.

(a) $\int_0^{\infty} x e^{-2x} dx$

(b) $\int_0^4 \frac{1}{\sqrt{4-y}} dy$

(c) $\int_{10}^{\infty} \frac{dx}{x^2 - 1}$

2. Suppose that $\int_a^b f(x) dx$ is approximated by TRAP(5) = 4.019 and TRAP(15) = 3.954. Use this to give a better estimate of $\int_a^b f(x) dx$.

3. Let $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$. Show that $\Gamma(n+1) = n\Gamma(n)$ and then use this to write $\Gamma(n)$ in terms of $\Gamma(n-1)$.

4. Determine whether $\int_3^{\infty} \frac{\cos x}{\sqrt{x^3+1}} dx$ converges or diverges. If it converges, approximate to three decimal places the number it approaches.

5. Evaluate

(a) $\int \frac{\ln x}{x} dx$ (b) $\int \frac{x-1}{\sqrt{x+1}} dx$ (c) $\int \sqrt{1+\sqrt{x}} dx$ (d) $\int x \ln x dx$ (e) $\int x\sqrt{1-x} dx$