

Show all relevant work!

YOU MAY USE A CALCULATOR TO VERIFY SOLUTIONS, BUT NOT TO PROVIDE THEM.

1. 900 cc/sec

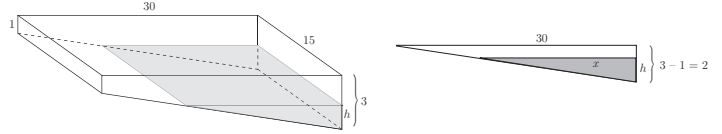
 Hint: $\frac{dx}{dt} = 3$ cm/sec., want $\frac{dV}{dt}$, and we know $V = x^3$ so $\frac{dV}{dx} = 3x^2$

 2. $\frac{1}{30}$ m/min or $\frac{10}{3}$ cm/min.

 Hint: Want $\frac{dh}{dt}$ and we know $\frac{dV}{dt} = 15$ m³/min.

 The volume of the pool for $h \leq 2$ is $V = \frac{1}{2}15 \cdot h \cdot x$.

 From the triangle, we have $\frac{x}{h} = \frac{30}{2} \rightarrow x = 15h$.

 So $V = \frac{225}{2}h^2$ and $\frac{dV}{dh} = 225h \rightarrow = 450$ at $h = 2$.

 3. Draining at $\frac{49\pi}{36}$ ft/min.

 Hint: Want $\frac{dV}{dt}$ and we know $\frac{dh}{dt} = -0.25$ ft/min.

 To relate V and h we know $V = \frac{1}{3}\pi r^2 h$ but we need to eliminate the r so use similar triangles: $\frac{r}{h} = \frac{7}{12} \rightarrow r = \frac{7}{12}h$.

 Then $V = \frac{1}{3}\pi \left(\frac{7}{12}h\right)^2 h = \frac{49\pi}{432}h^3$ and $\frac{dV}{dh} = \frac{49\pi}{144}h^2$.

 4. (a) $\frac{dP}{dt} = \frac{24}{\pi}$ in/sec.

 Hint: Want $\frac{dP}{dt}$ and we know $\frac{dC}{dt} = 6$ in/sec. To relate P and C note that the side of the square is equal to $2r$ so $P = 8r$. Then since $C = 2\pi r \rightarrow r = \frac{C}{2\pi}$ So $P = 8 \left(\frac{C}{2\pi}\right) = \frac{4C}{\pi}$. It follows $\frac{dP}{dC} = \frac{4}{\pi}$.

 (b) $\frac{dA}{dt} = \frac{120}{\pi} - 30$ in²/sec

 Hint: Want $\frac{dA}{dt}$ and we know $\frac{dC}{dt} = 6$ in/sec. To relate A and C note that the side of the square is equal to $2r$ so $A = (2r)^2$. Then since $C = 2\pi r \rightarrow r = \frac{C}{2\pi}$ So $A = \left(2 \cdot \frac{C}{2\pi}\right)^2 = \frac{C^2}{\pi^2}$. It follows $\frac{dA}{dC} = \frac{2C}{\pi^2}$. When $A = 25\pi \rightarrow r = 5$ so $C = 10\pi$.

 5. $\frac{10}{3}$ cm²/sec.

 Hint: Want $\frac{dA}{dt}$ and we know $\frac{dV}{dt} = 10$ cc/sec. To relate A and V note that both are given in terms of r :

 $A = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$ so $V = \frac{4}{3}\pi \left(\frac{A}{4\pi}\right)^{3/2}$. It follows $\frac{dV}{dA} = \frac{1}{2} \left(\frac{A}{4\pi}\right)^{1/2}$.

6. 800m of the \$1/m fence and 200m of the \$2/m fence. The cost will be \$1200.

 Hint: If x is the front and y represents the sides, we have (1) $x \cdot y = 60000$ and (2) $C = 2x + 1y + 1x + 1y$.

 Simplifying and substituing (1) we have $C = 3x + 2 \left(\frac{60000}{x}\right)$.

 7. $r \approx 3.56$ cm and $h \approx 8.91$ cm.

 Hint: What to minimize cost through surface area: $C = (0.03)\pi r^2 + (0.02)2\pi r h + (0.02)\pi r^2 = (0.05)\pi r^2 + 0.04\pi r h$.

 The additional constraint is $V = \pi r^2 h = 355$. Substituting the constraint for h gives $C = (0.05)\pi r^2 + 0.04\pi r \left(\frac{355}{r^2}\right)$.

 It follows we need to maximize $C = (0.05)\pi r^2 + \frac{14.2\pi}{r}$

8. (a) When $x = 1$, $A = \frac{1}{2}$.

Hint: Area of rectangle is given by base (x) and height ($y = \frac{1}{x^2+1}$) so $A = \frac{x}{x^2+1}$. Find max. of this function.

(b) No. IP at $x = \sqrt{\frac{1}{3}}$

Hint: Want y'' (NOT A''). Note $y'' = \frac{2(3x^2 - 1)}{(x^2 + 1)^3}$.

9. $r = \frac{20\sqrt{2}}{3}$ cm, $h = \frac{40}{3}$ cm.

Hint: If r is the radius of the base of the cone, then using the radius of the sphere = 10cm and Pythagoras, we have the height of the cone, $h = 10 + \sqrt{10^2 - r^2}$, assuming the cone extends below the equator.

Note: If $h < 10$, (so the base of the cone is above the equator) then we have $h = 10 - \sqrt{10^2 - r^2}$. Can you see why there is a cone with the same base area but $h > 10$ so we can ignore all cones where $h < 10$?

From this it follows $V = \frac{1}{3}\pi r^2(10 + \sqrt{10^2 - r^2})$.

10. We want to maximize $A = xy$ with the constraint that $2x + 2y = k$. $2x + 2y = k \rightarrow y = \frac{k-2x}{2}$.

Then $A = x\left(\frac{k-2x}{2}\right) = \frac{1}{2}kx - x^2$. So $\frac{dA}{dx} = \frac{k}{2} - 2x = 0 \rightarrow x = \frac{k}{4}$. Since $\frac{d^2A}{dx^2} = -2$, this is a maximum.

It follows that since $y = \frac{k-2(\frac{k}{4})}{2} = \frac{k}{4}$, the rectangle is a square.

11. (a) 3573.3' from the box along the road.

Hint: Want to minimize cost: $C = 35\sqrt{900^2 + x^2} + 20(4200 - x)$. Notice the domain bounds: $[0, 4200]$ and what they mean in this context. $x = 4200$ would be the shortest overall *distance*, while $x = 0$ would be the shortest path through the forest.

(b) \$109,850.53 (Note that $C(0) = \$115,500$)

(c) \$150,337.12 (when $x = 4200$)

12. 36π

13. $\frac{\pi}{4}$

14. (a) 20' for circle and 0' for square.

(b) $\frac{20\pi}{\pi+4}$ ' for circle and $\frac{80}{\pi+4}$ ' for square.

15. \$37