Exam 4 Name:

Show all relevant work!

Formulas:

Sphere	Cylinder	Elasticity of Demand	Exp. Growth/Decay
$V = \frac{4}{3}\pi r^3$	$V = \pi r^2 h$	$E = -\frac{dq}{dp} \cdot \frac{p}{q}$	$P = P_0 e^{kt}$
$S = 4\pi r^2$	$S = 2\pi r^2 + 2\pi r h$		

1. The table below shows the mean income in 2004 for U.S. workers based on years of school.

Years of School	6	10	12	13	14	16	18
Mean Income ($)	20,082	22,232	30,640	35,970	37,480	53,581	67,361

(a) Store these data in your calculator and use a cubic regression to model them. Write your function below.

$$I(t) = -10.377t^3 + 760.88t^2 - 9448.1t + 51540$$

(b) Use your model and some calculus to help you determine when the rate at which the mean income per year of education is *growing fastest*. (Within a reasonable domain).

$$I' = -31.131t^2 + 1521.76t - 9448.1$$

$$I'' = -62.262t + 1521.76$$

$$0 = -62.262t + 1521.76$$

$t = 24.44 \longrightarrow$ After 24 years of education (!)

(c) Use your model and some calculus to help you determine how many years of school maximizes your mean income. Comment on this result.

$$I' = -31.131t^2 + 1521.76t - 9448.1 = 0$$

$t = 7.29$ & $t = 41.57$

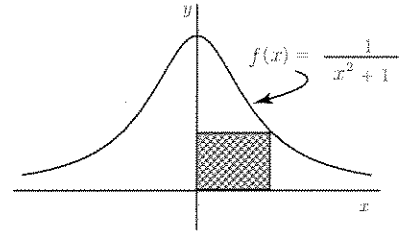
↑ MIN ↑ MAX

AFTER 41 years of school.
POSSIBLE BUT SEEMS UNLIKELY — SO BUSY BEING IN SCHOOL THAT BY THE TIME YOU'RE EMPLOYED, YOU'RE NOT MAKING $ FOR LONG.

2. A rectangle is inscribed beneath the function $f(x) = \frac{1}{x^2+1}$ so that it is bounded by the x and y axes (see below).

(a) Determine the area of the largest rectangle that can be inscribed this way.

(b) Does the x value of this rectangle occur at an inflection point of $f(x)$? (Explain)



(a) $A(x) = \dfrac{x}{x^2+1}$

$$A'(x) = \frac{(x^2+1) - x(2x)}{(x^2+1)^2}$$

$$= \frac{1-x^2}{(x^2+1)^2}$$

MAX WHEN $A'(x) = 0$

SO $1 - x^2 = 0$

$x = \pm 1 \to x = 1$

$$\boxed{A(1) = \frac{1}{2}}$$

(b) $f'(x) = \dfrac{-2x}{(x^2+1)^2}$

$$f''(x) = \frac{-2(x^2+1)^2 + 2x(2)(2x)(x^2+1)}{(x^2+1)^4}$$

$$= \frac{\cancel{(x^2+1)}(-2(x^2+1) + 8x^2)}{(x^2+1)^{\cancel{4}\,3}}$$

$$= \frac{6x^2 - 2}{(x^2+1)^3}$$

$= 0$ AT $x = \pm\sqrt{1/3} \neq 1$

SO NOT AT I.P.

(OR $f''(1) = 4/8 = \frac{1}{2} \neq 0$ SO NOT I.P.)

3. A balloon (in the shape of a sphere) is deflating. If air is leaking out at a rate of 7 cubic centimeters per second, how quickly is the radius of the balloon changing when its radius is 12cm?

$\dfrac{dV}{dt} = -7$ Find $\dfrac{dr}{dt}$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$

$$-7 = 4\pi r^2 \Big|_{r=12} \cdot \frac{dr}{dt}$$

$$\frac{-7}{4\pi \cdot 144} = \frac{dr}{dt}$$

$$\frac{dr}{dt} \approx -0.0039 \text{ cm/sec.}$$

4. The demand for yams is given by $q = 5000 - 10p^2$ where q is in pounds of yams and p is the price of a pound of yams in dollars.

(a) If the current price of yams is \$2 per pound, how many pounds will be sold?

$$q = 5000 - 10(2)^2$$
$$= 4960 \text{ lbs}$$

(b) For what price range will the demand for yams remain inelastic? Interpret what this means in terms of the demand for yams.

$$E = \frac{-dq}{dp} \cdot \frac{p}{q} = +20p \cdot \frac{p}{5000 - 10p^2}$$

$$= \frac{+20p^2}{5000 - 10p^2} < 1$$

$$+20p^2 < 5000 - 10p^2$$
$$p < \sqrt{\frac{500}{3}} \approx 12.91$$

For $p <$ \$12.91/lb.

As long as the price per lb. of yams stays under \$12.91, consumers will be willing to pay enough to balance the drop in demand.

Extra Credit \$5000 is deposited in an account where interest is compounded quarterly. If the balance of the account is growing at \$750.70/yr. when $t = 5$ years, what is the interest rate of the account? (you will need the solver on your calculator so just set up the equation for credit).

Trivia: Who is Norville Rogers better known as?

$$5000\left(1 + \frac{r}{4}\right)^{4t}$$

$$\frac{d}{dt}: \quad 5000\left(1+\frac{r}{4}\right)^{4t} \cdot \ln\left(1+\frac{r}{4}\right)^4 \cdot \frac{1}{4}$$

$$5000\left(1+\frac{r}{4}\right)^{4t} \cdot \ln\left(1+\frac{r}{4}\right) = 750.7$$